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Crack analysis and modeling in concrete beams reinforced with FRP composite sheets

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Abstract

In this article using the principles and relationships governing fracture mechanics and finite elements cracking in the first mode for reinforced concrete beams reinforced with FRP sheets are analyzed and modeled. In this method to simulate the crack in reinforced beams, the relations for determining the stress intensity coefficients with the presence of rebars and reinforcement sheets are developed. Here, it is assumed that a composite sheet is completely bound to the bottom surface of the beam under pure bending moment. In the proposed method beam components are divided into two categories, including components without cracks and components with cracks. In components without cracks, relations, equations, and the conventional stiffness matrix governing the beam are used, taking into account the changes in the moment of inertia caused by the presence of reinforcements and FRP sheets. In the finite component with a crack, the crack profile is simulated by creating a geometric defect in the beam section. So that the reduction in the hardness of the component with a crack is equivalent to the change in the dimensions of the discontinuity. Here, the changes in the hardness of the cracked component are calculated and presented as a function of the modified stress intensity coefficients. To ensure the correctness and accuracy of the presented method, all the analyzes are implemented in Abaqus software. The comparison of the obtained results shows that the presented method is a suitable method for the analysis of reinforced concrete structures resistant to cracking. So that it can be extended and developed for other models with proper accuracy. (DOI:**https//doi.org/10.52547/JCER.5.1.1**)

Keywords: crack; reinforced concrete beams; FRP sheets; stress intensity coefficients; Abaqus software

1. Introduction

Many cracked reinforced concrete structures suffer from basic damage and crack growth due to contact with corrosive factors and cyclic loading until finally the effects of complete failure and exit from access appear in them. This issue causes a lot of costs to repair, rebuild or replace damaged structures all over the world. So that millions of dollars are spent annually to repair and replace these structures. In an official statistic, the cost of repairing damaged rebar corrosion and cracking of reinforced concrete structures in the United States is estimated at 1 to 3 trillion dollars. In different regions of Iran, the destructive effects of crack growth and, as a result, the effect of corrosive factors in the foundations and

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beams of bridges, dams, reinforced concrete channels, etc., cause heavy costs for the restoration and reconstruction of these buildings. Using epoxy coating on steel parts and rebars, injecting polymer into concrete surfaces and cathodic protection of rebars are some effective techniques to prevent the growth of cracks and corrosion of steel in concrete. Research results show that each of these techniques has been only partially successful. Today, researchers have proposed the idea of replacing steel parts and steel bars with new materials that are resistant to stress concentration caused by cracking and erosion of reinforcements. The use of new materials, especially composites instead of steel, has been highly favored by researchers in the last decade. Composites consist of an adhesive material (mostly epoxy) and an appropriate amount of fibers. These fibers may be carbon, glass, aramid, etc., the composite obtained with these fibers is called AFRP, GFRP, CFRP respectively.

1.1. Cracking in concrete structures reinforced with rebar

One of the first studies on the modeling of reinforced concrete structures using the finite element method was done in 1973 by Krishnamoorthy [1] and by presenting a computer program to investigate the behavior of a reinforced concrete frame. Next, Zdenek et al [2] presented an analytical model based on the determination of structural stresses using previous methods to calculate the elasticity of cracked reinforced concrete beams. Since the nonlinear behavior of these structures has a special role in static and dynamic analysis, Hu and William [3] conducted one of the first studies on the nonlinear analysis of cracked reinforced concrete structures in 1989. A suitable and efficient model for modeling these beams is to use the torsion spring model. This method can even be generalized to model cracking in structures with fiber concrete reinforced with FRP sheets, which was presented by Krawczuk [4] in 1995. In this method, by determining the stress intensity coefficients, the stiffness of the torsion spring is determined and corrected. The use of stress intensity coefficients to model the behavior of reinforced concrete structures with cracks was strongly considered by researchers, so studies in this field have continued until now and extensive research has been done on this topic [11-5]. With the progress of science, many numerical methods based on Computer programming have been developed for modeling cracks in reinforced concrete structures and in the field of analyzing the behavior of these structures [12-16].

1.2. Cracking in concrete structures reinforced with FRP sheet

Due to the excellent resistance of FRP composites against corrosion and cracking, these materials have been highly regarded by researchers. So that there have been extensive studies on the modeling and analysis of cracked reinforced concrete beams with the application of FRP sheets [17]. These studies show that the bending strength and fatigue life of the cracked concrete beam increases greatly after strengthening with reinforcement sheets. In their studies, Biokosturik and Hernig [18] theoretically proved that the yield of concrete beams after strengthening by FRP sheets is significantly reduced. Recently, the research conducted by Ardini and Nani [19] prove that by using reinforcement sheets, the final bending capacity and the stiffness of concrete beams with a rectangular cross-section can be significantly increased. Tammy et al. [20] and Zhi and Gerstel [21] investigated and presented cracking propagation in a beam under three-point bending without strengthening methods. They proved that these theoretical predictions cannot be extended to a strengthened beam. Talgestin [22] and Terin Taflo [23] introduced methods to solve stress problems in FRP-reinforced concrete beams. They focused on determining the shear stresses and stresses that lead to the separation of the epoxy material. In these studies, it was found that the modulus of elasticity, thickness, and geometry of the reinforcement sheet affect the maximum shear stress and stress in the area of the glued sheet. In the current studies, if the crack is located in the middle of the beam, a common method for determining the tensile stresses of the sheet has not been provided. Therefore, it is necessary to provide a simple method to determine the stresses on the FRP sheet in order to evaluate the resistance of the sheet to prevent the propagation of cracks. Composites are widely used as a new advanced material in strengthening cracked concrete structures. While these materials are light in weight, they have high tensile strength and hardness. They also have high fatigue resistance against cyclic loading. Since concrete is a material with low tensile strength (about 1/10 of the ultimate compressive strength), the need for a reinforcing member to overcome this limitation is always felt. In this article, the effects of using FRP reinforcement members on the yield of cracked concrete beams are discussed by presenting a theoretical method. The use of composite reinforcement sheets causes geometrical changes in the beam section and consequently changes in the formulation for determining stress intensity coefficients and correcting the moment of inertia of cracked and uncracked sections. In this method, assuming the existence of a composite sheet on the lower surface of the beam, the relations of stress intensity coefficients are developed for this state. Using these modified stress intensity coefficients, crack modeling is done based on the creation of a geometric defect in the beam section. Finally, using a finite element method, the cracked reinforced concrete beam reinforced with FRP sheets is analyzed in static mode and its results are compared and verified with the simulations performed in Abaqus software.

2. An overview of previous research

One of the accurate and basic methods for analyzing beams with cracks is using the principles of fracture mechanics and determining stress intensity coefficients. In general, this method is based on the changes of strain energy and the changes of the second moment of the surface, before and after the creation of the crack. In this method, first, the geometric characteristics of the crack are simulated by reducing the cross-sectional area of the beam according to Figure (1). Then, the released energy rate is calculated due to the strain energy changes in the complete and cracked section.

In this method, the released energy is related to the values of stress intensity coefficients based on the relationship proposed by Erwin [25-24]. This coefficient for a beam with a rectangular section under pure bending is obtained from equation (1).In this method, the released energy is related to the values of stress intensity coefficients based on the relationship proposed by Erwin [25-24]. This coefficient for a beam with a rectangular section under pure bending is obtained from equation (1).

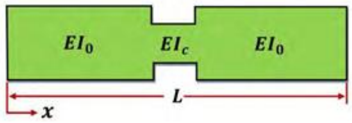

Figure 1- Reducing the cross-section of a cracked beam with changes in the moment of inertia

$$K_{I_M} = M(x)\sqrt{\frac{1}{b}\left(\frac{1}{I_0}-\frac{1}{I_C}\right)} \qquad \text{equation (1)}$$

where K_I is the stress intensity coefficient of the beam under bending moment, M is the bending moment, b is the thickness of the beam, E is the modulus of elasticity, I_0 is the moment of inertia of the cracked section and I_C is the moment of inertia of the cracked section.

By creating a deep link between the principles of fracture mechanics and the finite element method, this model can respond to the static, vibrational, buckling, and dynamic behaviour of composite structures with cracks. Since reinforced concrete structures are among the composite materials, as a result, the present method seems appropriate for the analysis of these structures. The relations presented by Kinzler [26] for calculating stress intensity coefficients were developed by Yokoyama [27-28] and Rice [29] as follows.

$$K_{I_M} = M\sqrt{\frac{1}{bI_0}\left(\frac{I_0}{I_C}-1\right)} \qquad \text{equation (1-2)}$$

$$K_{I_M} = \frac{6M}{bh^2}\sqrt{\pi a}F_M(\xi)\ ,0 \le \xi \le 0.6\ ,\xi = \frac{a}{h}$$

$$F_M(\xi) = \sqrt{\left(\frac{2}{\pi\xi}\right)tan\frac{\pi\xi}{2}}\frac{0.923+0.199\left[1-\sin\left(\frac{\pi\xi}{2}\right)\right]^4}{\cos\left(\frac{2}{\pi\xi}\right)}$$

equation (2-2)

$$K_{I_M} = \frac{3.99M}{bh\sqrt{h}\sqrt{(1-\xi)^3}}\ ,\ 0.6 < \xi < 1$$

equation (3-2)

where a is the depth of the crack, h is the height of the beam, and is defined as the ratio of the depth of the crack to the height of the beam.

Although extensive studies have been conducted on cracking in unreinforced beams and beams reinforced with steel reinforcements, limited research was done in the field of strengthening cracked beams by external reinforcements. A model for analyzing the problems of fracture mechanics of steel-reinforced concrete beams is the use of the stress intensity coefficients method, which was first presented by Carpentry [30]. In this method, the stress intensity factor in the reinforced concrete beam is determined by two independent factors. The stress intensity coefficient is caused by the external force (KM) and the stress intensity coefficient is caused by the tensile force of closing the rebars (Fs), which is denoted by (KF). As a result, the stress intensity factor for the cracked reinforced concrete beam reinforced with steel bars is presented as follows.

$$K_I = K_M - K_F \qquad \text{equation (3)}$$

3. Crack modeling formulation in reinforced concrete beams reinforced with FRP sheet

In this method, one-dimensional finite elements are used to model cracks in reinforced concrete beams reinforced with FRP sheets. The limited components of the beam are divided into two categories as shown below. The first category is components without cracks, which are simulated with finite elements equivalent to reinforced concrete and FRP sheet. These components are considered once using the transformed section method, integrated with the modulus of elasticity of concrete and by increasing the cross-sectional area at the place of the reinforcements. And once again, the cross-section of FRP sheet is equated with concrete and finally, they are considered as a single cross-section. The second category is the finite component with a crack, which is equated using a geometric defect. Due to the presence of a crack in this limited component, the cross-sectional area of the beam at the place of the crack decreases as much as the depth of the crack. The schematic representation of this category is presented in figure (2).

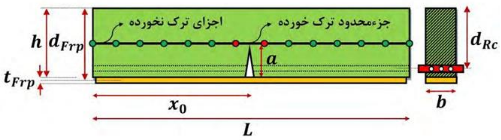

Figure 2- Cracked and uncracked finite elements in reinforced concrete beam reinforced with FRP sheets

3.1. Formulation of finite element method for non-cracked components with FRP sheet

The finite element method for non-cracked elements is investigated according to the Euler-Bernoulli beam theory. So that the basic equations can be defined by ignoring the effects of shear force and rotational inertia in the form of relations (4-1) and (4-2).

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\,\frac{\partial^2 w}{\partial x^2}\,,\ \frac{\partial u_0}{\partial x} = 0 \longrightarrow \varepsilon_x = -\,z\,\frac{\partial^2 w}{\partial x^2} \qquad \text{equation (4-1)}$$

$$\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0 \longrightarrow u = u_0 - z\,\frac{\partial w}{\partial x} \qquad \text{equation (4-2)}$$

where ε_x is the axial strain, u is the axial displacement, w is the vertical displacement and γ_{xz} is the shear strain.

According to equations (4-1), the structural equations for an elastic Euler-Bernoulli beam are defined as equation (5).

$$\sigma_x = E\varepsilon_x \longrightarrow \sigma_x = -E\,\frac{\partial^2 w}{\partial x^2} \qquad \text{equation (5)}$$

where σ_x is the axial stress and E is the modulus of elasticity of the beam.

In the Euler-Bernoulli beam theory, one-dimensional finite elements are used to analyze the finite elements of beams. The stiffness matrix of the beam without cracking is determined using the potential energy equation according to equations (4) and (6).

$$U = \frac{1}{2}\int \varepsilon_x{}^T \sigma_x dV \longrightarrow U = \frac{1}{2}\int_0^{l_e} EI_0 \left(\frac{\partial^2 w}{\partial x^2}\right)^T \left(\frac{\partial^2 w}{\partial x^2}\right) dx\,,\ w = [N]u \qquad \text{equation (4)}$$

where U is the potential energy, l_e is the length of the element and N is the function of the Hermitian shape according to []. By placing relation (5) in relation (4), the stiffness matrix of a limited component of the beam is obtained, which can be generalized for other components.

$$K_0 = EI_0 \int_0^{l_e} [N'']^T [N''] dx$$

equation (6-1)

$$K_0 = \frac{EI_0}{l_e^3} \begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4{l_e}^2 & -6l_e & 2{l_e}^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2{l_e}^2 & -6l_e & 4{l_e}^2 \end{bmatrix}$$

equation (6-2)

where K_0 is the stiffness matrix of a finite component of the beam without cracking.

What is important here is the changes in the moment of inertia caused by the presence of reinforcing armatures and FRP sheets in the structure. whose effects should be considered in relation (2-6). For this purpose, the idea of a transformed cross-section according to Figure (3) is used. In this method, the cross-section of the reinforced concrete beam with a compressive area above the neutral web, and a tensile area below the neutral web, equals n_1- 1 to the cross-sectional area of steel and n_2- 1 to the cross-sectional area of the FRP sheet. As the compressive strength of concrete is known, according to Iran's concrete regulations, the modulus of elasticity of concrete is determined according to equation (1-7).

$$E_c = \left(3300\sqrt{f_c} + 6900\right)\left(\frac{\gamma_c}{23}\right)^{\frac{3}{2}}$$

equation (7-1)

$$n_1 = \frac{E_s}{E_c}$$

equation (7-2)

$$n_2 = \frac{E_{Frp}}{E_c}$$

equation (7-3)

Here, n_1 and n_2 are the ratio of the elasticity coefficient of steel to concrete and the ratio of the elasticity coefficient of composite to concrete, respectively.

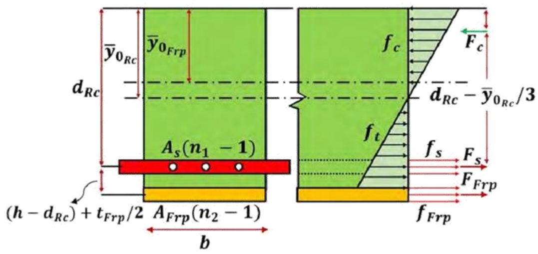

Figure 3- The transformed section of the reinforced concrete beam standard

According to Figure (3), the location of the neutral web and the modified moment of inertia in the beam reinforced with reinforced concrete and FRP sheet in the uncracked state are determined according to relations (1-8) to (3-8):

$$\bar{y}_{0_{Frp}} = \frac{bh\left(\frac{h}{2}\right) + \left(A_{Frp}(n_2 - 1)d_{Frp}\right)}{bh + A_{Frp}(n_2 - 1)}$$

equation (8-1)

$$\bar{y}_{0_{RC}} = \frac{bh\left(\frac{h}{2}\right) + \left(A_s(n_1 - 1)d_{RC}\right)}{bh + A_s(n_1 - 1)}$$

equation (8-2)

$$I_{0_{Frp}} = \frac{bh^3}{12} + bh\left(\frac{h}{2} - \bar{y}_{0_{RC}}\right)^2 + A_s(n_1 - 1)\left(d_{RC} - \bar{y}_{0_{RC}}\right)^2 + bh\left(\frac{h}{2} - \bar{y}_{0_{Frp}}\right)^2 + A_{Frp}(n_2 - 1)\left(d_{Frp} - \bar{y}_{0_{Frp}}\right)^2$$

equation (8-3)

where $\bar{y}_{0_{RC}}$ is the location of the neutral web in the reinforced concrete beam without cracks, $I_{0_{Frp}}$ is the moment of inertia of the reinforced concrete beam without cracks, A_s is the cross-section of the beams, d_{RC} is the distance between the reinforcements and the upper web of the beam, $\bar{y}_{0_{RC}}$ is the location of the neutral web in the beam reinforced with FRP sheets in the state without Crack, A_{Frp} is the cross-section of the composite sheet and D_{Frp} is the distance between the sheet and the upper web of the beam.

By inserting the equation (3-8) into the equation (2-6), the stiffness matrix of the components of the reinforced concrete beam without cracking is presented according to the equation (9).

$$K_{0_{Frp}}^{st} = \frac{EI_0}{l_e^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4{l_e}^2 & -6l_e & 2{l_e}^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2{l_e}^2 & -6l_e & 4{l_e}^2 \end{bmatrix}$$

equation (9)

3.2. Formulation of finite element method for the cracked finite element with FRP sheet

The implementation of the finite element method for the cracked (enriched) finite element is done according to the reduced cross-section method according to Figure (4).

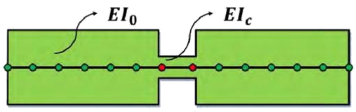

Figure 4- Reducing the cross-section of a reinforced concrete beam with a crack

By reducing the cross-section of the beam, the location of the neutral web and the moment of inertia of the reinforced concrete section at the crack location are shown in figure (5).

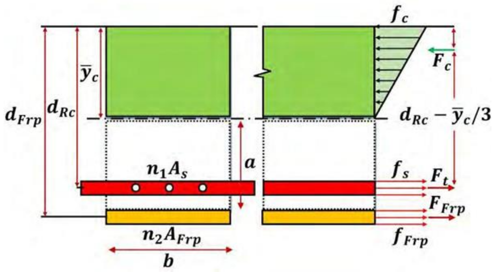

Figure 5- Cracked transformed section

As a result, by rewriting the relations (8-1) to (8-3), the location of the neutral web and the moment of inertia of the cracked reinforced concrete section in the state that they are reinforced with FRP sheet, are determined as relations (10-1) and (10-2).

$$\bar{y}_{C_{Frc}} = \frac{b(h-a)+\frac{h-a}{2}}{b(h-a)}$$

equation (10-1)

$$I_{C_{Frp}} = \frac{b\bar{y}_c^3}{3} + n_1A_s(d_{RC}-\bar{y}_C)^2 + n_2A_{Frp}(d_{Frp}-\bar{y}_C)^2$$

equation (10-2)

where $\bar{y}_c$ is the location of the neutral web in the reinforced cracked beam and $I_{C_{Frp}}$ is the moment of inertia of the reinforced cracked reinforced concrete beam.

By placing the moments of inertia extracted for healthy and cracked sections, according to relations (2-10) and (3-8) in relation (1), stress intensity coefficients for cracked reinforced concrete beam with composite reinforcement sheet are determined as relation (11).

$$K_{I_{Frp}} = \frac{M}{b}\frac{1}{\frac{bh^3}{12}+bh(\frac{h}{2}-\bar{y}_{0_{RC}})^2+A_s(n_1-1)(d_{RC}-\bar{y}_{0_{RC}})^2+bh(\frac{h}{2}-\bar{y}_{0_{Frp}})^2+A_{Frp}(n_2-1)(d_{Frp}-\bar{y}_{0_{Frp}})^2} - \frac{M}{b}\frac{1}{\frac{b\bar{y}_c^3}{3}+n_1A_s(d_{RC}-\bar{y}_c)^2+n_2A_{Frp}(d_{Frp}-\bar{y}_c)^2}$$

equation (11)

After determining the modified stress intensity coefficients, the crack-equivalent discontinuity can be replaced by a torsion spring. So that the reduction of the stiffness of the cracked area is equated with the reduction of stiffness and the increase of the torsional moment of the torsion spring. As a result, the degree of softness of the torsion spring is presented as a function of the stress intensity coefficient in the form of equation (12).

$$\lambda_M = \frac{1}{K_S} = \frac{2b(1-v^2)}{E}\int_0^a (\frac{K_{I_{Frp}}}{M})^2 da$$

equation (12)

where λ_M is spring softness, k_S is spring stiffness, v is Poisson's ratio and $K_{I_{Frp}}$ is the corrected stress intensity factor for the reinforced section. By inserting equation (10-2) into equation (6-2), the reinforced stiffness matrix of the cracked component of the reinforced concrete beam is presented according to equation (13).

$$K_{C_{Frp}} = \frac{EI_{Frp}}{l_{e_c}^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix}$$

equation (13)

where l_{e_c} is the length of the cracked area. The stiffness matrix of the standard reinforced concrete components in the reinforced state with the composite sheet is combined with the stiffness matrix of the cracked limited component, which finally forms the final stiffness matrix of the structure.

4. Discussion and results

In this section, to check the accuracy and correctness of the proposed model, the behavior of a reinforced concrete beam with a crack is tested in the form of a practical example. So that the changes in the depth and location of the crack on the elasticity of the elastic beam are investigated and its values are verified with Abaqus software.

4.1. Model specifications

A reinforced concrete beam with a crack and reinforced with a composite sheet with assumed geometric and material specifications according to Table (1) is examined under simple two-end boundary conditions. The crack in the tensile web of the beam is assumed to be constant along the thickness. Crack depth changes from zero to 6.0 height and crack location changes in the area between 0.05 – 0.95 beam length are investigated.

Table 1- Features of the reinforced concrete beam with cracks

$L = 4\,m$	$d_{Frp} = 0.3\,m$	$E_c = 2.5e10\,N/m^2$
$h = 0.35\,m$	$A_s = \pi D^2/4\,m^2$	$E_s = 2.0e11\,N/m^2$
$b = unit$	$A_{Frp} = 0.00113998\,m^2$	$E_{Frp} = 2.6e11\,N/m^2$
$t_{Frp} = 0.0011398\,m$	$\frac{a}{h} = 0.0 - 0.6$	$\upsilon_c = 0.25$
$D_{Rc} = 3\emptyset 22\,m$	$\frac{x_0}{L} = 0.05 - 0.95$	$\upsilon_s = 0.3$
$d_{Rc} = 0.27\,m$	$q = w \times L = 4e6\,N/m$	$\upsilon_{Frp} = 0.3$

4.2. Verification of crack depth changes

To check the accuracy and efficiency of the proposed model, a reinforced concrete beam reinforced with FRP sheet under the mentioned boundary and geometrical conditions is considered. First, by assuming the location of the crack in the middle of the beam to be fixed ($\frac{x_0}{L} = 0.5$), the deflection changes caused by the increase of the crack depth from zero to 0.6 height of the beam are investigated according to Figure (6) for the reinforced concrete beam under SS-SS boundary conditions.

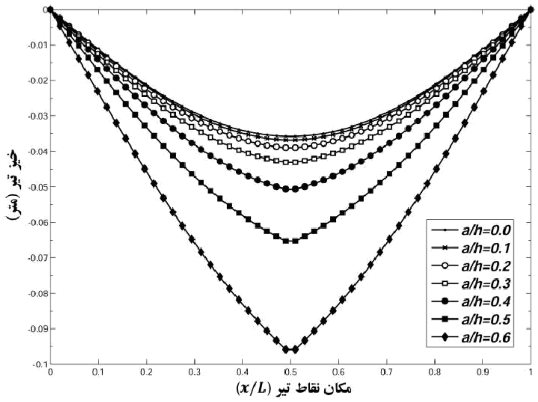

Figure 6- The effect of crack depth on the yield of the reinforced concrete beam (SS-SS)

To ensure the accuracy of the presented model, the obtained results are compared and validated with the simulation results in Abaqus software according to Table (2).

Table 2- Static analysis of reinforced concrete beam with a crack in the middle, under SS-SS boundary conditions with changes in crack depth

a/h	*Beam deflection* Article method (m)	*Beam deflection* Abaqus (m)	Error (%)
0	-0.03629	-0.03487	1.49929
0.1	-0.03705	-0.03589	2.329163
0.2	-0.03882	-0.03797	3.327761
0.3	-0.04436	-0.04426	5.115812
0.4	-0.04992	-0.04942	4.58037
0.5	-0.06409	-0.06164	1.581803
0.6	0.093524	-0.09341	5.506505

The results from table (2) show that up to a depth of 0.5 beam height, the modeling error is around 4%. The error growth at depths greater than 0.5 is caused by the nonlinear behavior of concrete and steel, the effects of which are not considered in this research. The validation results show the appropriate accuracy of the presented method in the static analysis of the elastic-reinforced concrete beam with cracks.

4.3. Checking the changes in the place of crack

In this review, the accuracy of the proposed model is checked by assuming a constant crack depth (a/h=0.3, 0.5) and changing the crack location between -0.05 to 0.95 beam length, according to Figure (7).

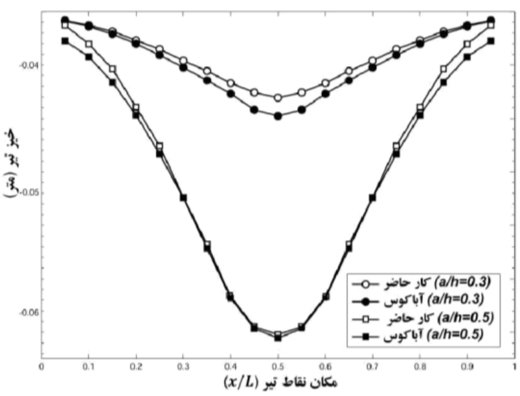

Figure 7- The effect of crack location on the deflection of the reinforced concrete beam (SS-SS)

Examining Figure (7) shows the appropriate accuracy of the presented method, so that it can be seen that the curves related to the results of finite element modeling and abacus completely coincide with each other. In this case, the reason for the error of 5.5% seems to be not considering the effects of the shear force in the assumptions of the Euler-Bernoulli beam theory, which is negligible.

5. Conclusion

In this article, the static behavior of the cracked reinforced concrete beam reinforced with FRP sheet has been analyzed and investigated. The analysis performed is based on the improvement of the finite element method and by dividing the beam components into two categories. The first category is the standard non-cracked components that are modified based on the modification of the moment of inertia caused by the presence of rebars and composite sheet with the converted section method. The other category is the finite element with a crack, which is modeled using the reduced section method. Using the provided relationships, the stiffness matrices of the standard and cracked components are modified and the final stiffness of the cracked structure is determined. In the current study, the effects of the depth and location of the crack on the yield of the beam were investigated and the results were validated. Comparison and analysis of the obtained results show:

A) The changes in rise caused by the depth of the crack in the range of less than 0.1 beam height have not been taken into account due to the lack of influence. Also, due to the non-linear behavior of concrete in a range greater than 0.6 beam height, the changes in creep due to the depth of the crack are not considered in this area.

B) The comparison of the obtained results with changes in crack location and depth showed that the proposed new solution for modeling cracks in reinforced concrete beams using the finite element method is acceptable with appropriate accuracy.

C) As the depth of the crack increases, the amount of deflection of the beam always increases. This increasing process gains more momentum from the depth of 0.4 height so that with small changes in the depth of the crack, the rise increases greatly.

References

[1] Krishnamoorthy, C., Panneerselvam , S. A. (1978). FEP-ACSl-a finite element program for nonlinear analysis of reinforced concrete framed structures, Journal of Computer and Structures, Vol. 9, pp. 451–461J. Newman, Electrochemical Systems, 2nd ed., Prentice-Hall, Englewood Cliffs, NJ, 1991.

[2] Zdenek, P.B., Byung, H. (1984). Deformation of Progressively Cracking Reinforced Concrete Beams, Journal of ACI, Vol. 71, pp. 268–278.B. Miller, Proc. 6th Australian Electrochem. Conf., Geelong, Vic., 19-24 Feb., 1984; J. Electroanal. Chem., 168 (1984) 91.

[3] Hu, H., William, C. (1990). Nonlinear analysis of cracked reinforced concrete, Journal of ACI, Vol. 87, pp. 199–207.

[4] Krawczuk, M., Ostachowicz, W. M. (1995). Modelling and vibration analysis of a cantilever composite beam with a transverse open crack, Journal of Sound and Vibration, Vol. 183, pp. 69–89.

[5] Husnu, M., Alaattin, A. (2000). Analytical and finite element comparisons of stress intensity factors of composite materials, Journal of Composite Structures, Vol. 50, pp. 99–102.

[6] Lau, K., Shi, S. (2001). Estimation of stress intensity factor (KI) for an FRP bonded concrete beam using the superposition method, Journal of Concrete Research, Vol. 53, pp. 31–41.
[7] Yang, Q. G., Chen, Q. (2004). Analysis of the stress intensity factor of vertical cracks in small-size beams bonded with GFRP, Journal of Damage and Fracture Mechanics, Vol. 55, pp. 67–79.
[8] Wang, K., Inman, D., Farrar, C. (2005), Modeling and analysis of a cracked composite cantilever beam vibrating in coupled bending and torsion, Journal of Sound and Vibration, Vol. 284, pp. 23–49.
[9] Ju, S. H., Liu, S. H. (2007), Determining stress intensity factors of composites using crack opening displacement, Journal of Composite Structures, Vol. 81, pp. 614–621.
[10] Mogadpalli, G. P., Parameswaran, V. (2008), Determination of stress intensity factor for cracks in orthotropic composite materials using digital image correlation, Journal of compilation, Vol. 44, pp. 446– 452.
[11] Dotti, F., Cortnez, V., Reguera, F. (2013), Mode I stress intensity factor for cracked thin-walled composite beams, Journal of Theoretical and Applied Fracture Mechanics, Vol. 68, pp. 38–45.
[12] Rogoza, A., Ubysz, A. (2009), Numerical analysis of the crack reainforced concrete beams, 18th International Conference on the Application of Computer Science and Mathematics in Architecture and Civil Engineerings, Germany, 07-09 July.
[13] Benarbia, D., Benguediab, M., Benguediab, S. (2013), Two-dimensional Analysis of Cracks Propagation in Structures of Concrete, Journal of Engineering, Technology & Applied Science Research, Vol. 3, pp. 429– 432.
[14] Subramani, T., Manivannan, R., Kavitha, M. (2014), Crack Identification in Reinforced Concrete Beams Using Ansys Software, Journal of Engineering Research and Applications, Vol. 4, pp. 133–141.
[15] Slowik, M., Smarzewski, P. (2014), Numerical Modeling Of Diagonal Cracks In Concrete Beams, Journal of Archives Of Civil Engineering, Vol. 3, pp. 307–322.
[16] Costa, D., Carmo, R., Costa, R. (2016), Numerical modeling of concrete beams under serviceability conditions with a discrete crack approach and noniterative solution-finding algorithms, Journal of International Federation for Structural Concrete, Vol. 18, pp. 225–236.
[17] Saadatmanesh, H., Ehsani, M. R. (1990), Fibre composite plates can strengthen beam, Journal of the American Concrete Institute, Vol. 12, pp. 65-71.
[18] Buyukozturk, O., Hearing, B. (1998), Failure behaviour of precracked concrete beams retrofitted with FRP, Journal of Composites for Construction, Vol. 3, pp. 138-144.
[19] Arduini, M., Nanni, A. (1997), Behaviour of precracked RC beams strengthened with carbon FRP sheets, Journal of Composites for Construction, Vol. 1, pp. 63-69.
[20] Toumi, A., Bascoul, A., Turatsinze, A. (1998), Crack propagation in concrete subjected to flexural cyclic loading, Journal of Materials and Structures, Vol. 31, pp. 451-458.
[21] Xie, M., Gerstle, W. H., (1995), Energy-based cohesive crack propagation modeling, Journal of Engineering Mechanics, Vol. 121, pp. 1349-1358.
[22] Taljsten, B. (1997), Strengthening of beams by plate bonding, Journal of Materials in Civil Engineering, Vol. 9, pp. 206-212.
[23] Triantafillou, T. C. (1998), Shear strengthening of reinforced concrete beams using epoxy-bonded FRP composites. ACI Structural Journal, Vol. 95, pp. 107-115.
[24] Irwin, G.R., Kies, J.A. (1954), Critical energy rate analysis of fracture strength, Journal of Welding, Vol. 33, pp. 193-198.
[25] Irwin, G.R. (1957), Analysis of stresses and strains near the end of a crack traversing a plate, Journal of Applied Mechanics, Vol. 24, pp. 361-364.
[26] Kienzler, R., Herrmann, G. (1986), An elementary theory of defective beams. Journal ofActa Mechanica,Vol. 62 , pp. 37-46.
[27] Yokoyama, T. (1996), Vibration analysis of Timoshenko beam-columns on two-parameter elastic foundations, Journal of Computers & Structures, Vol. 61, pp. 995-1007.
[28] Yokoyama, T., Chen, M.C. (1998), Vibration analysis of edge-cracked beams using a linespring model, Journal of Engineering Fracture Mechanics, Vol. 59, pp. 403-409.
[29] Ricci, P., Viola, E. (2006), Stress intensity factors for cracked T-sections and dynamic behavior of T-beams, Journal of Engineering Fracture Mechanics, Vol. 73, pp. 91–111.
[30] Carpinteri, A. (1986), Mechanical Damage and Crack Growth in Concrete, Martinus Nijhoff, Dordrecht.