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Optimal Design of a Composite Trapezoidal Channel Cross Section Using Sensitivity Analysis

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ABSTRACT

This paper presents the optimal shape of a compound trapezoidal cross-section by considering sensitivity analysis of design variable in shape optimization of trapezoidal open channel. the sensitivity analysis of the effect of discharge and channel bottom slope on design variables, constraints and objective function. The objective function is to minimize the cost of excavation and lining and the cost of water lost as seepage and evaporation losses, the design variables are the depth and width of the channel bottom, side slopes and free board, the constraints include uniform flow, maximum and minimum velocity, Froud number and water surface width. Optimization was performed with different values of discharge and channel bottom slope in two stages and with several models. In the first stage, at a constant slope, with increasing discharge, the values of discharge, bed width, free board, flow velocity, flow cross-sectional area and cost increase, but the side slope decreases. in the second stage, at constant discharge values, as the slope of the channel bottom increases, the values of flow depth, bed width, flow cross-section area, and overall channel construction cost decrease. Results of sensitivity analysis show that all of design variables are important.

ARTICLE INFO

Received: June 11 , 2026
Accepted: July 01 , 2026

Keywords:
Open channels
Optimal design
Cost of water loss
sensitivity analysis
Composite Trapezoidal Channel

DOI: 10.66224/JCER.8.3.28
DOR: 20.1001.1.22516530.1399.11.4.1.1

1. Introduction

The optimal design of the cross section of the channel is necessary for the design discharge to convey through it and the least cost for its construction [1]. Structural shape sensitivity analysis, that is, the calculation of quantitative information on how the response of a structure is affected with respect to changes in its parameters that define its shape, plays a key role in shape optimization [2]. Sensitivity analysis can be used to study how the uncertainty in the output of a mathematical model or system (numerical or otherwise) can be apportioned to different sources of uncertainty in its inputs [3]. For a successful optimization the requirements are a good finite element model, adequate sensitivities, proper choice of objective function, design variables, constraints and a suitable method of solution of the nonlinear mathematical problem [4]. The optimal design of the composite channel cross-section was first addressed by Trout (1982) with the aim of minimizing lining costs with composite roughness

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for trapezoidal, rectangular, and triangular channels. [5]. Guo, and Hughes, (1984) for the first time considered the free board as a design variable for the optimal design of the trapezoidal channel using the first principles of calculus and derivation and with the objective function of frictional resistance or the construction cost [6]. In Monadjemi's 's research (1994), the triangular section is the best hydraulic section among different sections such as rectangular, triangular, trapezoidal and round bottom triangular [7].

Swamee (1995) and Swamee et al. (2001, 2002, 2002) Presented optimal channels with triangular, rectangular, trapezoidal, and circular sections, considering water lost as seepage and evaporation losses, and using presented a non-linear method. The results showed that the cross-sectional area of the channel and losses due to seepage and evaporation in a trapezoidal channel are less than triangular and rectangular channels. [8-13]. The results of Babaeyan-Koopaei et al.'s research (2000) showed that the cross-sectional the wetted flow area and wetted perimeter of a triangular channel with a parabolic bottom is less than a channel with a triangular and parabolic cross-section. [14]. Das (2000) presented the optimal shape of the cross-section with composite roughness trapezoidal channel with freeboard using the method of Lagrange coefficients, taking into account the constraints of Manning's equation and the objective function of the construction cost. [15]. Jain et al. (2004) presented the optimal design of the trapezoidal channel section by considering the velocity and top width constraints of the water surface using genetic algorithm. The cost of construction a channel using genetics algorithms is lower compared to the method of Lagrange multiplier. [16]. Bhattacharjya (2005) considered problem for optimal design of composite channel cross section of trapezoidal shape by investigating the constraint of freeboard and changes in the specific energy of the channel. [17]. Bhattacharjya (2006) optimized the trapezoidal channel section with nonlinear optimization model incorporating the critical flow condition of the channel. The results show that this method has a better and more favorable performance compared to the Lagrange method in the research of Das. [18]. Also, Bhattacharjya used the optimal shape of the open channel cross-section with the combined method of genetic algorithm and second-order sequential programming algorithm. The results showed the high efficiency of the mentioned method in the optimal and stable design of open channels. [19]. Das (2007) presented the optimal cross-section model of a trapezoidal channel with two objective functions of minimizing the overall cost of channel construction and minimizing the probability of overtopping and limiting the establishment of uniform flow using the first-order analysis method. The results of this research showed that with the decrease of overtopping probability, the flow depth decreases and the bed width increases. [20]. Bhattacharjya and Satish (2008) developed the model of Das (2007) by considering the freeboard as a design variable and also using genetic algorithm. The results indicate a direct relationship between the width of the channel and the cost of its construction, as well as the better efficiency of the genetic algorithm compared to the classical optimization. [21]. Reddy and Adarsh (2010) presented the optimal shape of the cross-section of the composite trapezoidal channel using the elitist-mutated particle swarm optimization (EMPSO) method with overtopping constrained design. The results show that the cost of construction the channel is lower than the method of Lagrange coefficients. [22]. In recent years, Easa (2011), Easa and Vatankhah, (2014), Han (2015), Han and Easa (2017), very extensive researches regarding the design of new general open channel cross-sections were introduced. The results show that the optimal section that minimizes construction cost is substantially better than the most hydraulically efficient section. [23-24]. Pourbakhshian and Ghaemian (2015) presented a methodology to determine the optimal shape of double curvature concrete arch dams and the sensitivity analysis of design variables in shape optimization of concrete arch dams. [25]. Orouji et al. (2016) optimized the trapezoidal compound channel section using the frog leaping algorithm. The results show that the use of this algorithm is more economical compared to genetic algorithms, ants, etc. [26]. Roushangar et al. (2018) in their studies to provide the optimal shape of the composed trapezoidal channel using genetic algorithm showed that the application of depth, speed and Froud number constraints increases the construction cost, while the water surface width constraint reduces the cost. [27]. Gupta et al. (2018) used Fish Shoal Optimization (FSO) for the optimal design of the trapezoidal channel section. which reduced the cost of channel construction compared to particle swarm algorithm (PSO) [28]. Farzin and Anarki (2020) using meta-heuristic methods, to the optimal and probabilistic design of the composed trapezoidal channel cross-section in which models are based on constant or variable freeboard, uniform or composite roughness coefficient, fixed and variable freeboard, and also velocity, Froude number and overtopping constraints using bat hybrid algorithm. The results showed that using HBP, compared to BA, PSO, LINGO, Lagrange multiplier method and shuffled frog-leaping algorithm, led to a 32% reduction in the cost of channel construction. [29]. Pourbakhshian and Pouraminian (2021) presented some analytical models for the optimal design of trapezoidal composite channel cross-section. The objective function is the cost function per unit length of the channel, which includes the excavation and lining costs. The results are presented in three parts. In the first part, the optimal values of the design variables and the objective function are presented in different discharges. In the second part, the relationship between cost and design variables in different discharges is presented in the form of conceptual and analytical models and mathematical functions. Finally, in the third part, the changes in the design variables and cost function are presented as a graph based on the discharge variations. Result indicate that the cost increases with increasing water depth, left side slope, equivalent roughness coefficient, and freeboard. [30]. the optimal cross section of a composite parabolic channel by considering four models based on freeboard changes. The results show that increasing the discharge increases the flow depth, left and right side slopes, total top width and water surface width, channel cross-sectional area and flow area, the total channel perimeter and wetted perimeters, flow velocity, Froud number and the cost increases. [31].

the optimal shape of the compound trapezoidal cross-section is presented by considering the deterministic constraints and the probabilistic constraint of channel flooding as uncertainty with the SPSA algorithm and with three flow discharge of 10, 50, and 120 m3/s. The cost of construction and the bed width, initially decreases with the increase of the overtopping probability, and at a certain value, the probability reaches its minimum value, and then with the increase of the overtopping probability, the cost of construction and bed width increases. [32].

The purpose of this paper is to determine the optimal shape of the cross-section of the composite trapezoidal channel and also sensitivity analysis of design variables in shape optimization of trapezoidal open channels. In the optimization process, the objective function is the cost canal sections. Coefficients sensitivity analysis is performed based on the 10000 trails by utilizing Latin Hypercube Sampling. Uniform distributions have been assigned for simulations. Correlation analysis is used to determine relationships between design variables and objective function. The program basically consists of two parts that are described in the paper: The first section recalls the optimization procedure of the cross section of a composite trapezoidal channel. In Section 2, sensitivity analysis is performed.

2. Optimal design of open channels

2.1. Trapezoidal cross section

Figure (1) shows the cross section of a composite trapezoidal channel.

As shown in Figure 1, z_1:1,z_2:1represent the side slopes of the channel corresponding to the left and right sides , n_1,n_2,n_3 are Manning's roughness coefficients on the left, right and bottom sides of the channel, b is bottom width, y is the flow depth, f is the freeboard, and s_0 is the longitudinal bed slope.

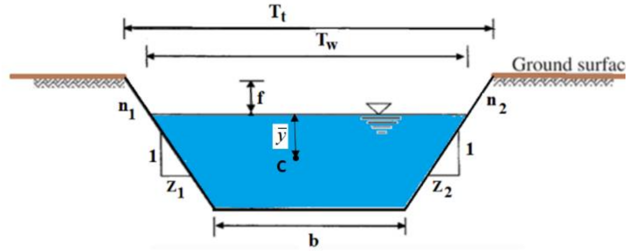

Figure 1. Representation of the cross-sectional geometry of the composite trapezoidal channel [32]

where A_t and P_t are the is the total channel cross-sectional area and perimeter, respectively; T_tis the total top width of the channel cross-section; A_wand P_ware the channel wetted area and perimeter, respectively; T_wis the water surface width,1 P_w1, P_w2, and P_w3 are wetted perimeters corresponding to the left, right and bottom sides of the channel respectively. P_t1, P_t2, and P_t3are perimeters corresponding to the left, right and bottom sides of the channel, respectively; R_w is the hydraulic radius; and Dis the hydraulic depth. Parameters y, b, z1, z2, and f are defined in Figure 1 are related to the area (Equation 1), perimeter (Equation 2), the overall width of the section (Equation 3) and depth of centroid of area for trapezoidal canal sections. (Equation 4)

$$A_t = b(y+f) + (z_1 + z_2)\,(y+f)^2/2 \tag{1}$$

$$P_{t1} = ({z_1}^2 + 1)^{(1/2)}.\,(y+f)$$

$$P_{t2} = ({z_2}^2 + 1)^{(1/2)}.\,(y+f)$$

$$P_{t3} = b$$

$$P_t = P_{t1} + P_{t2} + P_{t3} \tag{2}$$

$$P_t = \left\{\left[({z_1}^2+1)^{(1/2)} + ({z_2}^2+1)^{(1/2)}\right](y+f) + b\right\}$$

$$T_t = b + (z_1 + z_2)(y+f) \tag{3}$$

$$\bar{y} = \frac{y}{6}\left(\frac{3b+2zy}{b+zy}\right) \tag{4}$$

In the above equations, by removing the freeboard in the equations, the corresponding water flow equations are obtained. In this paper, the left and right-side slopes of the channel are considered equal to each other.

2.2. Design variable

According to the geometric model described in figure (1), design variables, description and their minimum and maximum values are given in Table 1.

Table 1.
Representation of design variables

X_i	unit	Design variables	X_L	1
$x_1 = y$	m	Depth of flow	0.5	15
$x_2 = b$	m	Channel bed width	2	15
$x_3 = z$	-	Channel side slope	0.2	3
$x_4 = f$	-	Free board	0.5	2

2.3. Constraint2

Table 2.
Representation of design variables

Equation	Describe
$\Phi_1 = \left[1 + 2.457A_*\sqrt{R_*}\ln(\frac{\varepsilon_*}{12R_*} + \frac{0.221\upsilon_*}{{R_*}^{1.5}}\right] - \varepsilon_0$	Uniform flow
$\Phi_2 = \frac{F_r}{F_{r_{max}}}$	Sub Critical flow
$\Phi_3 = \frac{T_t}{T_{max}}$	Channel top width
$\Phi_4 = \frac{V_{min}}{V_{ave}}$	Minimum permissible velocity
$\Phi_5 = \frac{V_{ave}}{V_{max}}$	Maximum permissible velocity

According to Table 2, the constraint of uniform flow is considered to conduct the uniform flow in the channel [9]. Therefore, in this research, the Manning's equation constraint Horton's method was used to control uniform flow and to calculate the equivalent roughness coefficient [33].

Froud number constraint is included in order to avoid development of critical flow in the optimal design of the channel [14] . And the Channel top width is included to control the cost of land acquisition [15, 16, 22]. The minimum allowed velocity to prevent sedimentation is in the range of 0.6 to 0.9 m/s and the minimum velocity to prevent the growth of vegetation is 0.75 m/s [1]. The minimum velocity allowed in channel design is in the range of 0.75 to 0.9 [9]. In channels with a rigid boundary, the maximum allowable velocity (VL) is the velocity that does not cause erosion. Moreover, to ensure the conveyance of the discharge through the cross-section, the mean actual flow velocity in the channel should not exceed the maximum permissible velocity [11] and [16]. In this study, the minimum and maximum velocity values are 0.75 and 4 m/s, respectively.

2.4. Objective function

The optimal design of an open canal consists of minimization of an objective function which is subjected to certain constraints. The objective function consists of the cost per unit length of the canal.

In this paper, the total cost includes the earthwork cost, cost of lining, cost of water lost as seepage and evaporation subject to uniform flow condition in the canal.

2.4.1. Lining Cost

The lining cost depends on the extent of canal surface area to be lined and the type of lining material to be used. [35]

$$C_L = c_l P_t \quad (5)$$

C_L = cost of channel lining is per unit of channel length.

c_l= unit cost of lining.

2.4.2. Earthwork Cost

Earthwork is required for providing open canal flow area. Earthwork cost is the major cost item for a canal passing through hard and firm strata, where lining may not be required. (33)

Where C_e= Excavation cost is per unit of channel length

$$C_e = c_e A + c_r A\bar{y} \quad (6)$$

c_e= excavation cost per unit cross-sectional area for a unit length of the channel.

c_r=additional cost per unit volume of excavation per unit depth.

$\bar{y}$= depth (m) of the centroid of the area of excavation from the ground surface. (equation 4)

2.4.3. Cost of water lost as seepage and evaporation

One of the most important water lost in a canal is water seepage from the canal.

The seepage loss from a canal in a homogeneous and isotropic porous medium when the water table is at very large depth was written by Swamee et al. (2000) as:

$$q_s = k.y.F_s \quad (7)$$

qs = water loss as seepage per unit length of canal (m^2 /s).

k = hydraulic conductivity of the porous medium (m/ s).

y_n = normal depth of water in the canal (m);

Fs = seepage function (dimensionless), which is a function of the channel geometry.

$$F_s = \left[((4\pi - \pi^2)^{1.3} + (2z)^{1.3})^{\frac{0.77+0.468z}{1.3+0.6z}} + (\frac{b}{y})^{\frac{1+0.6z}{1.3+0.6z}}\right]^{\frac{1.3+0.6z}{1+0.6z}} \quad (8)$$

In equation (18), z is side slope, y is depth of flow and b is bed width.

evaporation of water from the channel surface is related to the energy source to provide the latent heat of evaporation and the ability to transport water vapor away from the evaporation surface, which depends on wind speed and relative humidity.

The evaporation loss was written as:

$$q_E = E.T_w \quad (9)$$

q_E = water loss as evaporation per unit length of canal (m^2 /s);

T = width of free surface (m)

E = evaporation discharge per unit surface area (m/s).

From the sum of equations (7) and (8), the water loss due to seepage and evaporation per unit of canal are obtained according to the following equation:

$$q_w = k.y_n.F_s + ET_w \quad (10)$$

$$C_w = \frac{3.156\times10^7 c_w}{r}(ky_nF_s + ET_w) \quad (11)$$

C_w = water loss cost ($\frac{\$}{m}$)

c_w= cost per unit volume of water (\$/m3).

r = rate of interest (\$/\$/year).

$$C_w = c_{ws}y_nF_s + c_{wE}T_w \quad (12)$$

c_{ws}= Cost of water loss due to water seepage from the canal($\frac{\$}{m}$)

c_{wE} =Cost of water loss due to water leakage from the canal($\frac{\$}{m}$)

$$c_{ws} = \frac{3.156\times10^7 kc_w}{r} \quad (13)$$

$$c_{wE} = \frac{3.156\times10^7 c_wE}{r} \quad (14)$$

Adding (5), (6), and (12), canal per unit length C (\$/m) was obtained as

$$C = C_E + C_L + C_w = c_lP_t + c_eA + c_rA\bar{y} + c_{ws}y_nF_s + c_{wE}T_w \quad (15)$$

Since $\frac{c_L}{c_e}, \frac{c_e}{c_r}, \frac{c_{ws}}{c_e}, \frac{c_{wE}}{c_e}$ have length dimensions. they remain unaffected by the monetary unit chosen. These ratios can be obtained for various types of linings, soil strata, and climatic condition by using appropriate unit rates[9,10].

The resistance equation is defined as follows:

$$V = -2.457\sqrt{gRS_0}\, ln(\frac{\varepsilon}{12R} + \frac{0.221\upsilon}{R\sqrt{gRS_0}}) = 0 \quad (16)$$

where V = average flow velocity (m/s); g = gravitational acceleration (m/s^2); R = hydraulic radius (m), defined as the ratio of the flow area to the flow perimeter; S_0= longitudinal canal bed slope (dimensionless); ε_0= average roughness height of the canal lining (m); and υ = kinematic viscosity of water (m^2/s). Using the continuity equation and (16), the discharge Q (m^3/s) was obtained as:

$$Q = AV = -2.457A\sqrt{gRS_0}\, ln(\frac{\varepsilon}{12R} + \frac{0.221\upsilon}{R\sqrt{gRS_0}}) = 0 \quad (17)$$

Since a canal is designed to sustain uniform flow, (17) provides the required condition as an equality constraint function in the design

$$Q + 2.457A\sqrt{gRS_0}\, ln(\frac{\varepsilon}{12R} + \frac{0.221\upsilon}{R\sqrt{gRS_0}}) = 0 \quad (18)$$

Uniform flow constraint is written as follows:

In order to easily determine the effect of variables and parameters of cost functions in the process of optimizing the trapezoidal channel shape, the equations are converted to dimensionless form by choosing a length scale.

L= length scale

$$L = (Q/\sqrt{gS_0})^{0.4} \quad (19)$$

Therefore, the nondimensional variables are defined as follows:

$$\bar{y}_* = \frac{\bar{y}}{L}, y_{n_*} = \frac{y_n}{L}, T_* = \frac{T}{L}, R_* = \frac{R}{L} \quad (20)$$

$$A_* = \frac{A}{L^2},\ P_* = \frac{P}{L}$$

$$C_* = \frac{C}{c_eL^2}, c_{l_*} = \frac{c_l}{c_eL}, c_{r_*} = \frac{c_rL}{c_e}, c_{ws_*} = \frac{c_{ws}}{c_eL}, c_{wE_*} = \frac{c_{wE}}{c_eL},$$

$$\varepsilon_* = \frac{\varepsilon}{L}, \upsilon_* = \frac{\upsilon L}{Q}$$

where subscript * refers the corresponding nondimensional parameter. Using (15), (18), and (20), the objective function in nondimensional form becomes as follows:

$$C_* = A_* + c_{r_*}A_*\bar{y}_* + c_{l_*}P_* + c_{ws_*}Fsy_{n*} + c_{wE_*}T_* \quad (21)$$

$$\Phi_{11} = 1 + 2.457A_*\sqrt{R_*}\, log(\frac{\varepsilon_*}{12R_*} + \frac{0.221\upsilon_*}{R_*^{1.5}}) = 0 \quad (22)$$

$$\Phi_1 = 1 + 2.457A_*\sqrt{R_*}\, log(\frac{\varepsilon_*}{12R_*} + \frac{0.221\upsilon_*}{R_*^{1.5}}) - \varepsilon_0 \quad (23)$$

In this paper,$\varepsilon_0$value equal to 0.01 was considered

2.5. optimization algorithm

The SPSA algorithm is a powerful algorithm for the optimization of complex systems that was developed and expanded by SPAL in 1998.[35] Among the features of the SPSA algorithm is that in each optimization iteration, regardless of the number of design variables, it only needs to evaluate the objective function twice. Therefore, the use of this algorithm greatly reduces the volume of calculations and reduces the total optimization time. [36] SPSA algorithm has been effectively used in civil engineering, especially in arch dam optimization. [37-40]

SPSA algorithm is for optimization of unconstrained problems. Therefore, in order to use it in solving bounded problems with unequal constraints, it is necessary to replace the objective function with the pseudo-objective function obtained by using the method of external penalty functions:

$$w(X,r_p) = f(X) + r_p \sum_{j=1}^{m} \max[0, g_j(X)]^2$$
$$\Rightarrow f(\bullet) = w(\bullet) + \text{noise} \qquad (24)$$

The above steps can be seen in the flowchart of figure (2).

The SPSA algorithm is used for optimizing unconstrained problems. Therefore, to use it for solving constrained problems with inequality constraintsg_j≤0" "(j=1,...,m), it is necessary to replace the quasi-objective functionw obtained by the external penalty function method with the objective function (f):

$$w(X,r_p) = f(X) + r_p \sum_{j=1}^{m} \max[0, g_j(X)]^2$$

$$\Rightarrow f(\bullet) = w(\bullet) + \text{noise} \qquad (25)$$

r_p is a penalty multiplier . The flow chart of the SPSA algorithm for the channel optimization problem can be shown in Figure 2.

2.6. Sensitivity analysis

Sensitivity analysis measures the changes in output by varying the inputs. the latter generated by sampling from their values distributional range. SA is therefore important to identify the key factors affecting the output [41].

In this section, the shape optimization of trapezoidal open channel is analyzed by design sensitivity analysis.

The model procedure is as follows:

1. Generation of a sample of design variables by SIMLAB software
2. By applying generation data, the basis for sensitivity analysis is derived by MATLAB software.

2.6.1. Generation of design variables

SIMLAB provides a free development framework for Sensitivity and Uncertainty Analysis [42].

To use SIMLAB the user performs the following operations.

2.6.1.1. Select a range and a distribution (probability distribution functions, pdfs) for each input factor.

These selections will be used in the next step for the generation of as ample from the input factors. In this paper, uniform (Rectangular) distribution is used.

If it is only possible to define a minimum and maximum value for a factor, a uniform distribution is appropriate. For a uniform distribution all values between the minimum and maximum values are equally likely to be sampled. It is the simplest of all continuous distributions. Parameters are the upper and lower bounds: a and b. The uniform PDF is given as [43]:

$$f(x) = \begin{cases} 0 & x \notin (a,b) \\ \frac{1}{b-a} & x \in (a,b) \end{cases} \qquad (26)$$

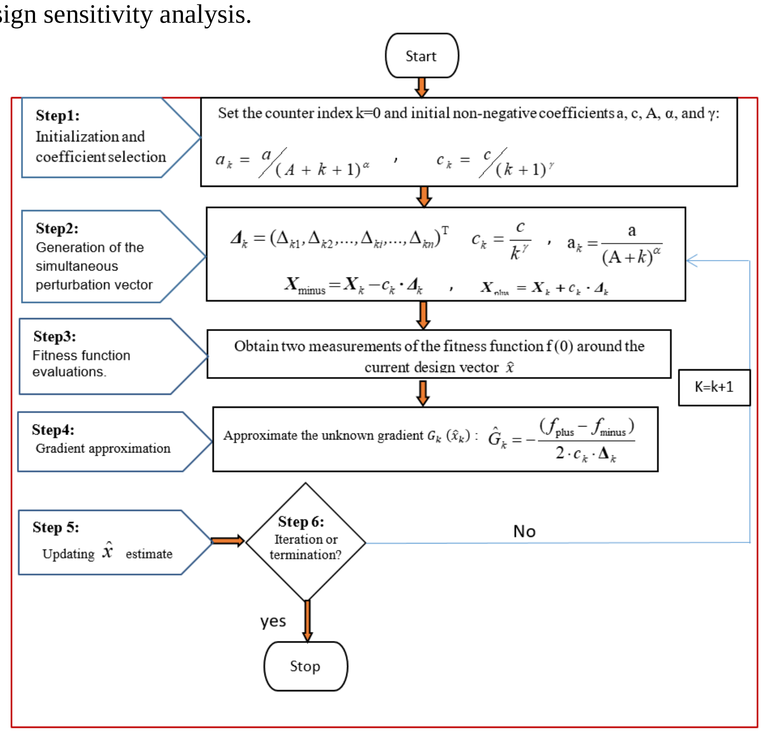

Figure 2. The flow chart of SPSA algorithm [32]

2.6.1.2. Generate a sample of elements from the distribution of the inputs previously specified.

The result of this step is a sequence of sample elements. Among different sampling strategies, The LHS is a particular case of stratified sampling which is meant to achieve a better coverage of the sample space of the selected input parameters [3]. The first step in the sample generation phase is to select range sand distributions (probability distribution functions, pdfs) for the input factors. The second step is to select a sampling method from among the available ones. The third step is the actual generation of the sample from the input pdfs. 1000 different samples have been generated with the help of Simlab. In this paper, For the generation LHS was used.

2.6.2. Extracting the results of sensitivity analysis by MATLAB software by applying the produced data

Correlation coefficients, which are the special cases of regression-based analysis, are typically useful methods for sensitivity analysis [3]. The Pearson correlation coefficient, for instance, can be used to characterize the degree of linear relationship between the output values and sampled values of individual inputs.

A quantitative estimate of linear correlation can be determined by calculating a simple correlation coefficient on the parameter values of input and output. Pearson's product moment correlation coefficient is denoted by r and is defined as:

$$r = \frac{\sum_{j=1}^{n}(X_{ij}-\overline{X_i})(Y_{ij}-\bar{Y})}{\left[\sum_{j=1}^{n}(X_{ij}-\overline{X_i})^2 \sum_{j=1}^{n}(Y_{ij}-\overline{Y_i})^2\right]^{\frac{1}{2}}} \quad (27)$$

X_{ij} are the values of the generated random parameters, $\overline{X_i}$ the average of these values, and Y_{ij} are the values of the output parameter, $\overline{Y_i}$ the average of these values, and n is the total number of generated data.

For the correlation between Xi and Y [44]. The larger the absolute value of r the stronger the degree of linear relationship between the input and output values. A negative value of r indicates the output is inversely related to the input. A linear regression on the data can be used to determine the correlation coefficient from the square root of the coefficient of determination, R^2 [45].

$$\text{Opt. Design vector}=\begin{bmatrix} D.V_1 \\ D.V_2 \\ . \\ . \\ . \\ DV_n \end{bmatrix}_{1\times n}$$

n=number of design variables.
N=number of samples

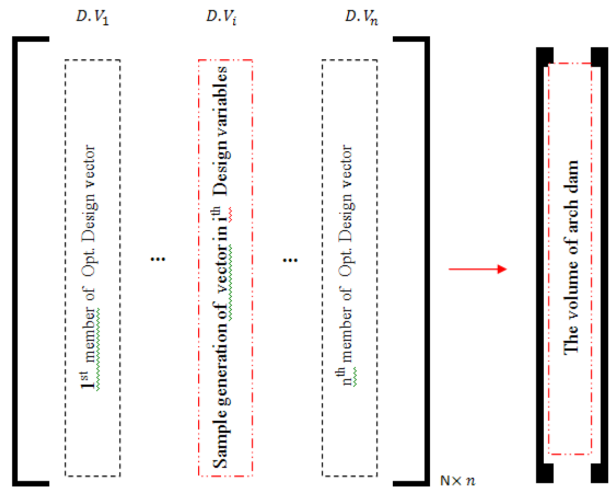

Figure 3. S.A. assessment flowcharts in trapezoidal composite open channel

$$r_n = \frac{\sum_{j=1}^{N}\left[(D.V_{nj}-\overline{D.V}_{nj})\,(C_j-\bar{C})\right]}{\left[\sum_{j=1}^{N}(D.V_{nj}-\overline{D.V}_{nj})^2 \sum_{j=1}^{N}(C_j-\bar{C})^2\right]^{\frac{1}{2}}} \quad (28)$$

In this paper, $D.V_{nj}$ are design variable (y,b,z,f), C_j is cost.

$S.A = r =$ Correlation coefficients

$$S.A_{index}=\begin{bmatrix} S.A_1 \\ S.A_2 \\ . \\ . \\ . \\ S.A_n \end{bmatrix} \quad (29)$$

3. Results and discussion

3.1. Result of optimization

In this article, design steps of the optimal trapezoidal composite channel cross-section have been done by considering the cost of seepage and evaporation in different values of discharge using the SPSA optimization algorithm. The discharge values considered from 50 to 500 m3/s respectively. These steps are as follows:

Step1: Swamee’s method is used to verify the results was used to compare the results of the SPSA algorithm.

The specifications of the model's input constant parameters are as follows:

$Q = 100\, {m^3}/{s}$ ، $S_0 = 0.001$ ، $\upsilon = 1.1 \times 10^{-6}\, m^2/s$ ، $\varepsilon = 1,\ g = 9.79 m/s^2$

The values of the parameters of the excavation and lining cost and the seepage and evaporation cost are as follows :(Swamee,2000):

$c_e = 1, \quad \frac{c_l}{c_e} = 12, \quad \frac{c_e}{c_r} = 7, \quad \frac{c_{ws}}{c_e} = 10, \quad \frac{c_{we}}{c_e} = 2$

It should be noted that in the model of Swamee (2000), only uniform flow constraint is specified in the optimization process.

Step2: Then, the model was developed with five very important constraints including, uniform flow constraint, Froud number constraint to control the subcritical flow in the channel, maximum velocity constraint to prevent scouring in the channel, minimum velocity constraint to control sedimentation in the channel and limiting the channel top width were investigated to reduce the cost of the land area in order to provide the optimal shape of the channel section.

In Table 3, the first and second rows respectively correspond to the results of the validation of the model with the SPSA algorithm with the Swamee (2000) model, which only includes the condition of uniform flow, and the third row corresponds to the results of the SPSA algorithm with the five constraint are definite.

Table 3.
Comparison of the optimization results of the trapezoidal composite channel section with the result of Swamee (2000)

Parameters / Model	y (m)	b(m)	Z	Fr	A (m^2)	V (m/s)	COST
Swamee (2000)	3.878	5.829	0.512	0.6	30.304	3.3	429.27
SPSA model	3.672	5.877	0.513	0.65	28.506	3.48	431.75

Comparing the results of the current model with the results of the result of Swamee (2000) shows that the SPSA algorithm has a very good convergence in reaching the desired solution.

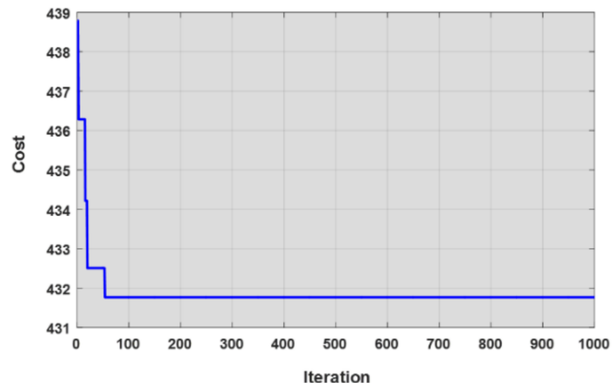

Figure 4. Convergence rate of the cost using SPSA algorithm for discharge 100 m3/s and bed slope S=0.001

Step3: After verifying and ensuring the correctness of the model in the first and second steps, the optimal shape of the compound trapezoidal channel section was obtained at different discharges with bed slope S=0.001.According to Table 5, increasing the discharge, increases the flow depth, bed width, freeboard, cross-sectional area of flow, water surface width, flow velocity and cost and decrease bed slope, Froud number.

Figures 5 show the changes in the optimal values of flow depth, channel bed width, side slope of the channel, and channel construction cost with different flow discharge.

According to Table 7, 8 and 9, increasing the discharge, increases the flow depth, bed width, freeboard, cross-sectional area of flow, water surface width, flow velocity and cost and decrease bed slope, Froud number.

Table 4.
Results from optimization in different runs using the SPSA algorithm with S0=0.001, Q=50 m3/s.

Y(m)	4.66	4.42	4.54	4.49	4.71	4.67	4.52	4.37	4.72
B(m)	6.35	6.37	6.22	5.21	5.52	5.72	6.39	6.5	5.5
Z	0.5	0.64	0.59	0.86	0.67	0.65	0.59	0.67	0.68
f	0.42	0.41	0.44	0.42	0.54	0.6	0.6	0.58	0.59
A_w(m2)	40.61	40.68	40.67	40.93	40.98	40.87	41.14	41.24	41.41
T_w(m)	11.06	12.05	11.66	13.01	11.89	11.76	11.8	12.37	11.99
V(m/s)	1.22	1.22	1.22	1.21	1.22	1.22	1.23	1.22	1.23
Fr	0.204	0.212	0.209	0.219	0.211	0.208	0.209	0.214	0.211
Cost1	273.2	273.9	274.7	279.1	281.4	283.5	284.2	285	285.7
Cost2	271.1	271.9	271.6	275.6	273.6	272.8	273	274.1	275.1
Cost	544.2	545.9	546.3	554.6	555	556.3	557.2	559.1	561.1

Table 5.
Presentation the values of design variables, constraints and Cost function at different discharges with a bed slope S0=0.0001

Q(m^3/s)	50	100	200	300	400	500
Y(m)	4.19	5.28	8.06	10.45	12.26	12.55
B(m)	6.58	8.53	10.46	11.62	13.67	16.31
Z	0.9	0.89	0.52	0.37	0.26	0.23
f	0.304	0.31	0.4	0.41	0.48	0.88
Fr	0.22	0.23	0.21	0.2	0.19	0.19
A_w(m^2)	43.42	70.07	118.04	162.09	204.55	242.33
T_w(m)	14.12	18	18.81	19.38	19.71	22.28
V(m/s)	1.22	1.41	1.68	1.83	1.94	2.04
Cost1	285.39	385.03	548.87	708.69	875.86	1028
Cost2	284.92	362.06	462.38	546.42	620.05	669.31
Cost	570.32	747.09	1011.26	1251.1	1495.9	1697.4

Table 6.
Presentation of conceptual models based on the dependent variable and independent variables

$$y^+, b^+, z^-, f^+, T^-, \rightleftarrows A^-, Fr^- \propto Cost^+$$
$$y^-, b^-, z^+, f^-, T^+, \rightleftarrows A^+, Fr^+ \propto Cost^+$$
$$y^-, b^+, z^+, f^-, T^+, \rightleftarrows A^+, Fr^+ \propto Cost^+$$
$$y^+, b^-, z^+, f^+, T^-, \rightleftarrows A^-, Fr^- \propto Cost^+$$

Table 7.
Presentation the values of design variables, constraints and Cost function at for flow discharge of 50 m3/s and bed slope S0=0.0001

S	0.0001	0.0004	0.0007	0.0010	0.0013	0.0016
y(m)	4.19	3.98	3.22	3.03	2.81	2.62
b(m)	6.58	5.96	4.32	4.02	3.91	3.85
z	0.90	0.52	0.74	0.49	0.52	0.55
f	0.304	0.290	1.540	0.280	0.270	0.290
Tw	14.12	10.16	9.06	7.03	6.81	6.75
V(m/s)	1.22	2.28	2.67	2.96	3.28	3.55
Aw (m^2)	43.42	32.09	21.57	16.70	15.08	13.93
Fr	0.22	0.41	0.50	0.61	0.70	0.79
Cost1(Cw)	285.39	232.35	241.79	159.54	150.62	145.17
Cost2(Cel)	284.92	240.71	198.76	173.79	165.15	158.71
Cost	570.32	473.06	440.56	333.35	315.76	303.88

3.2. Result of sensitivity analysis

After optimizing the composite trapezoidal channel cross-section, a sensitivity analysis of the effect of each design variable on the total cost was determined, which is shown in table 10 and Figure 6,7. Positive values of S.A indicate that when the design variables increase, the objective function, which is the total cost, also increases, and negative values of S.A mean that as the values of the design variables increase, the value of the objective function decreases. The S.A positive value means that when the design variables value increase, the objective function (total cost value increases, and the SA negative value means that when the design variables value increase, the objective function value decreases. All of the design variables are important.

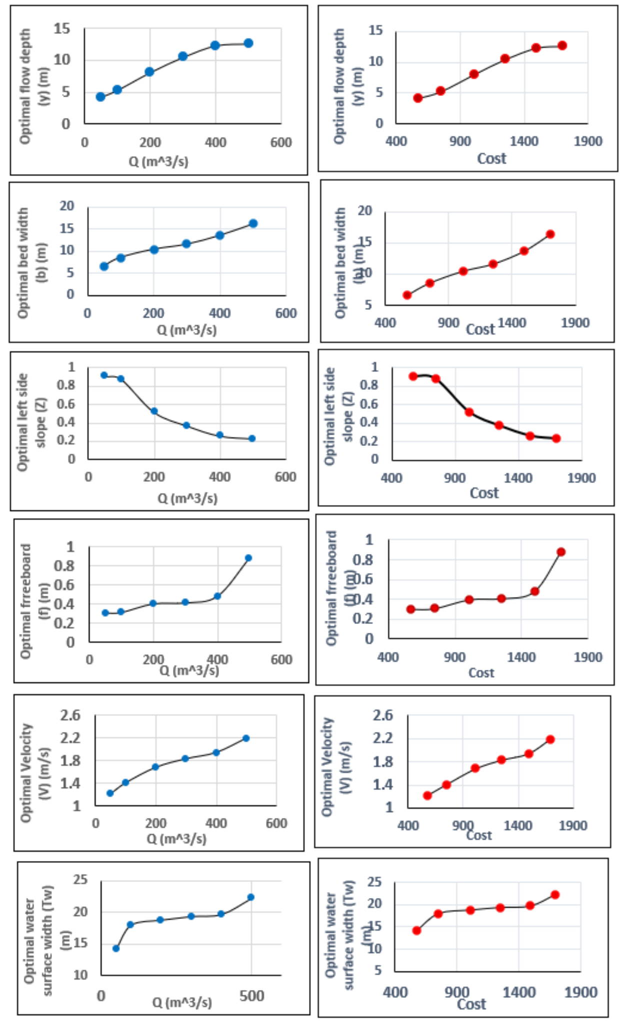

Figure 5. Results of optimization of composed trapezoidal cross-section using SPSA algorithm for different flow discharge values

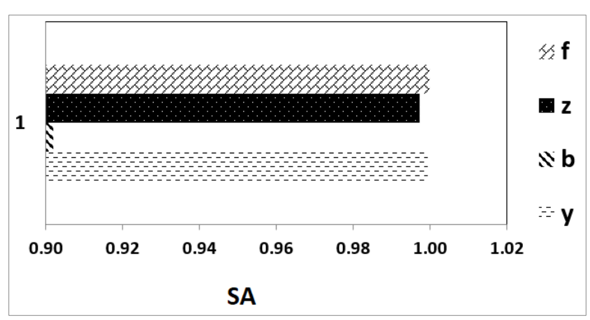

Figure 6. sensitivity indices for the cost of composite trapezoidal channel at the flow discharge of 50 m3/s

Table 8.
Presentation the values of design variables, constraints and Cost function at for flow discharge of 100 m3/s and bed slope S0=0.0001

S	0.0001	0.0004	0.0007	0.0010	0.0013	0.0016
y(m)	4.19	3.98	3.22	3.03	2.81	2.62
b(m)	6.58	5.96	4.32	4.02	3.91	3.85
z	0.90	0.52	0.74	0.49	0.52	0.55
f	0.304	0.290	1.540	0.280	0.270	0.290
Tw	14.12	10.16	9.06	7.03	6.81	6.75
V(m/s)	1.22	2.28	2.67	2.96	3.28	3.55
Aw (m^2)	43.42	32.09	21.57	16.70	15.08	13.93
Fr	0.22	0.41	0.50	0.61	0.70	0.79
Cost1(Cw)	285.39	232.35	241.79	159.54	150.62	145.17
Cost2(Cel)	284.92	240.71	198.76	173.79	165.15	158.71
Cost	570.32	473.06	440.56	333.35	315.76	303.88

Table 9.
Presentation the values of design variables, constraints and Cost function at for flow discharge of 200 m3/s and bed slope S0=0.0001

S0	0.0001	0.0004	0.0007	0.001	0.0013	0.0016
Y(m)	8.06	7.02	5.88	6.08	5.02	4.92
B(m)	10.46	9.52	8.06	7.22	5.94	5.91
z	0.52	0.63	0.52	0.56	0.55	0.47
f	0.40	0.21	0.97	0.30	0.27	0.82
Tw	18.81	18.30	14.22	13.20	11.80	10.55
V(m/s)	1.68	3.18	3.74	4.09	4.52	4.90
Aw(m^2)	118.04	97.64	65.57	49.31	43.77	40.47
Fr	0.23	0.46	0.54	0.65	0.74	0.79
Cost1(Cel)	548.87	467.50	397.79	306.2	279.13	290.88
Cost2 (Cw)	462.38	421.09	344.33	298.84	282.15	271.47
Cost	1011.3	888.59	742.14	605.08	561.47	562.37

Table 10.
sensitivity analysis of the input factors at different discharges

Design Variable	**SA (Q= 50 m^3/s)**	**SA (Q= 200 m^3/s)**
y	0.99947695	0.99944068
b	0.90191052	0.99995244
z	0.99735134	0.99786043
f	0.99995576	0.99995603

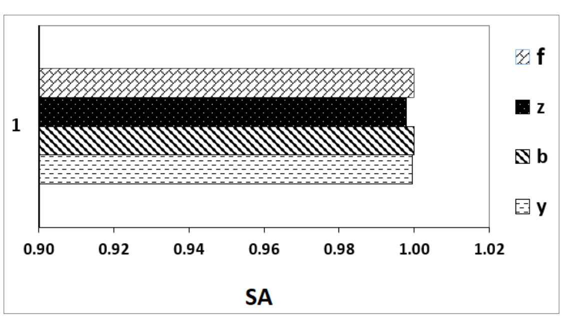

Figure 7. sensitivity indices for the cost of composite trapezoidal channel at the flow discharge of 200 m3/s

4. Conclusion

In this study, a methodology to determine the optimal shape of a composed trapezoidal open channel and the sensitivity analysis of design variables in shape optimization of composed trapezoidal channel are presented.

The effect of discharge is investigated by considering different values in these models on the design variables, constraints, objective function and geometric parameters and channel design. The results show that increasing the discharge, increases the flow depth, bed width, freeboard, cross-sectional area of flow, water surface width, flow velocity and cost and decrease bed slope, Froud number. Coefficients of sensitivity analysis are performed based on the 10000 trials by utilizing Latin Hypercube Sampling. Uniform distributions have been assigned for simulations. Correlation analysis is used to determine relationships between design variables and objective function. Results show that all of design variables are important.

References

[1] Chow, V. T. "Open channel hydraulics." New York: McGraw-Hill. (1959)

[2] Kwak, B. M. "A review on shape optimal design and sensitivity analysis." Journal of structure mechanic earthquake engineering, 10.4 (1994): 1-16. https://doi.org/10.2208/jscej.1994.483_1

[3] Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D., Saisana, M. and Tarantola, S." Global sensitivity analysis." the primer. John Wiley & Sons. (2008).

[4] Mota Soares, C. M., Mota Soares, C. A., & Barbosa, J. I. (1994). "Sensitivity analysis and optimal design of thin shells of revolution." AIAA journal, 32.5 (1994) :1034-1042

[5] Trout, T.J. 1982. "Channel design to minimize lining material costs. " Journal of Irrigation and Drainage Engineering, 108.4 (1982): 242-249. https://doi.org/10.1061/JRCEA4.0001392

[6] Guo, CY and Hughes, WC. "Optimal channel cross section with freeboard. " Journal of Irrigation and Drainage Engineering, 110 (1984): 304–314. https://doi.org/10.1061/(ASCE)0733-9437(1984)110:3(304)

[7] Monadjemi, P. "General Formation of Best Hydraulic Channel Section. " Journal of irrigation and drainage engineering, 120.1 (1994): 27-35. https://doi.org/10.1061/(ASCE)0733-9437(1994)120:1(27)

[8] Swamee, P. K. "Optimal irrigation canal sections. " Journal of irrigation and drainage engineering, vol. 121 (1995): 467-469. https://doi.org/10.1061/(ASCE)0733-9437(1995)121:6(467)

[9] Swamee, P. K. G. C. Mishra, and B. R. Chahar. "Comprehensive Design of Minimum Cost Irrigation Canal Sections. " Journal of irrigation and drainage engineering, vol. 126 (2000):322-327. https://doi.org/10.1061/(ASCE)0733-9437(2000)126:5(322)

[10] Swamee, P. K. G. C. Mishra, and B. R. Chahar. Minimum cost design of lined canal sections. " Water Resources Management, vol.14 (2000). 10.1023/A:1008198602337

[11] Swamee, P.K., Mishra, G.C., Chahar, B.R. "Design of minimum seepage loss canal sections with drainage layer at shallow depth. Journal of irrigation and drainage engineering,127.5, (2001): 287–294. https://doi.org/10.1061/(ASCE)0733-9437(2001)127:5(287)

[12] Swamee, P.K., Mishra, G.C., Chahar, B.R. "Design of minimum water loss canal sections. " J. Hyd. Res., IAHR, 40.2 (2002a): 215–220.https://doi.org/10.1080/00221680209499864
[13] Swamee, P.K., Mishra, G.C., Chahar, B.R. "Optimal design of a transmission canal. " Journal of irrigation and drainage engineering, 128.4, (2002b):234–243. https://doi.org/10.1061/(ASCE)0733-9437(2002)128:4(234)
[14] Babaeyan-Koopaei K, Valentine EM and Swailes DC. "Optimal design of parabolic-bottomed triangle. " Journal of Irrigation and Drainage Engineering, 126.6 (2000): 408–411. https://doi.org/10.1061/(ASCE)0733-9437(2000)126:6(408)
[15] Das A. "Optimal channel cross section with composite roughness. " Journal of irrigation and drainage engineering, 126.1 (2000):68-72. https://doi.org/10.1061/(ASCE)0733-9437(2000)126:1(68)
[16] Jain, A., Bhattacharjya, R.K. and Sanaga, S.. "Optimal design of composite channels using genetic algorithm. " Journal of Irrigation and Drainage Engineering, 130.4 (2004): 286-295. https://doi.org/10.1061/(ASCE)0733-9437(2004)130:4(286)
[17] Bhattacharjya, R.K. "Optimal design of open channel section considering freeboard. " ISH Journal of Hydraulic Engineering, 11.3 (2005): 141-151.https://doi.org/10.1080/09715010.2005.10514808
[18] Bhattacharjya, R.K. "Optimal design of open channel section incorporating critical flow condition. " Journal of irrigation and drainage engineering, 132.5 (2006): 513-518. https://doi.org/10.1061/(ASCE)0733-9437(2006)132:5(513)
[19] Bhattacharjya, R.K and M.G, Satish. "Optimal design of a stable trapezoidal channel section using hybrid optimization techniques. " Journal of irrigation and drainage engineering, 133.4 (2007): 323-329. https://doi.org/10.1061/(ASCE)0733-9437(2007)133:4(323)
[20] Das, A. "Flooding probability constrained optimal design of trapezoidal channels. " Journal of irrigation and drainage engineering, 133.1 (2007):53-60.https://doi.org/10.1061/(ASCE)0733-9437(2007)133:1(53)
[21] Bhattacharjya, R.K. and M, Satish.. "Flooding probability-based optimal design of trapezoidal open channel using freeboard as a design variable. " Journal of irrigation and drainage engineering, 134.3 (2008): 405-408. https://doi.org/10.1061/(ASCE)0733-9437(2008)134:3(405)
[22] Reddy, M. J., & Adarsh, S. "Chance constrained optimal design of composite channels using meta-heuristic techniques. " Water resources management, 24.10 (2010): 2221-2235. https://doi.org/10.1007/s11269-009-9548-5
[23] Easa, S. M., & Vatankhah, A. R. "New open channel with elliptic sides and horizontal bottom. " KSCE Journal of Civil Engineering, 18.4 (2014): 1197-1204. https://doi.org/10.1007/s12205-014-0559-2
[24] Han, Y. C., & Easa, S. M. "New and improved three and one-third parabolic channel and most efficient hydraulic section. " Canadian Journal of Civil Engineering, 44.5 (2017): 387-391. https://doi.org/10.1139/cjce-2016-0535
[25] Pourbakhshian, S., and M. Ghaemain. "Shape optimization of arch dams using sensitivity analysis." KSCE Journal of Civil Engineering 20.5 (2016): 1966-1976. https://doi.org/10.1007/s12205-015-0135-4
[26] Orouji, H., Mahmoudi, N., Fallah-Mehdipour, E., Pazoki, M., & Biswas, A. "Shuffled frog-leaping algorithm for optimal design of open channels. " Journal of irrigation and drainage engineering, 142.10 (2016): 06016008. https://doi.org/10.1061/(ASCE)IR.1943-4774.0001059
[27] Roushangar, K., Alami, M. T., Nourani, V., & Nouri, A. "A cost model with several hydraulic constraints for optimizing in practice a trapezoidal cross section. " Journal of Hydroinformatics, 19.3 (2017): 456-468. https://doi.org/10.2166/hydro.2017.081
[28] Gupta, S. K., Mishra, U., Datta, D., & Singh, V. P. "Fish shoal optimization for identification of the most suitable revetment stone for design of minimum cost earthen canals carrying sediment-laden flow." ISH Journal of Hydraulic Engineering, 24.2 (2018): 172-189.https://doi.org/10.1080/09715010.2017.1402211
[29] Farzin, S., & Valikhan Anaraki, M. "Optimal construction of an open channel by considering different conditions and uncertainty. " application of evolutionary methods. Engineering Optimization, 1-19. 2020)). https://doi.org/10.1080/0305215X.2020.1775825
[30] Pourbakhshian, S., and Pouraminian, M. "Analytical models for optimal design of a trapezoidal composite channel cross-section. " Civil and Environmental Engineering Reports, 31.1(2021): 118-138. https://doi.org/10.2478/ceer-2021-0009
[31] Pourbakhshian, S., and Fasih, P. "Optimization of composite parabolic channel cross-section based on changes in discharge and freeboard. "Sharif Journal of Civil Engineering (SJCE), 39.1(2022): 3-13. [In Persian] https://doi.org/10.24200/J30.2022.58880.3011
[32] Pourbakhshian, S., and Abdolrasuli, f. " Investigating the effect of uncertainty in the optimal design of a trapezoidal channel. " Journal of Civil Engineering Researchers 66.4 (2024) :40-51. https://doi.org/10.61186/JCER.6.4.40
[33] Horton, R. E. "Separate roughness coefficients for channel bottom and sides. " Engineering News-Record, 111.22 (1933): 652-653.
[34] Swamee, Prabhata Kumar, and B. R. Chahar. Design of canals. Springer India:., 2015.
[35] Spall, J. C. "An overview of the simultaneous perturbation method for efficient optimization. " Johns Hopkins apl technical digest, 19.4 (1998):482-492.
[36] Seyedpoor, S. M., Salajegheh, J., Salajegheh, E., & Gholizadeh, S. "Optimal design of arch dams subjected to earthquake loading by a combination of simultaneous perturbation stochastic approximation and particle swarm algorithms. " Applied Soft Computing, 11.1 (2011): 39-48.https://doi.org/10.1016/j.asoc.2009.10.014
[37] Pourbakhshian S. & Ghaemain M. "Shape optimization of arch dams using sensitivity analysis. " KSCE Journal of Civil Engineering, 20.5 (2016): 1966-76.https://doi.org/10.1007/s12205-015-0135-4
[38] Pourbakhshian, S., Ghaemain, M., & Jogatae, A. "The shape optimization of concrete arch dams considering stage construction. " Scientia Iranica, 23.1 (2016): 21-35. https://doi.org/10.24200/SCI.2016.2094
[39] Pouraminian, M., & Pourbakhshian, S. "SPSA Algorithm based optimum design of longitudinal section of bridges. " Indian Journal of Science and Technology, 7.9 (2014): 1327-1332. https://doi.org/10.17485/ijst/2014/v7i9/59472
[40] Pouraminian, M., & Pourbakhshian, S. "Shape Optimization of Concrete Arch Dams with SPSA Algorithm and Post-analysis of Pareto Front for selection the Best Alternatives. " Journal of Hydraulic Structures, 6.3 (2020): 59-82. https://doi.org/10.22055/JHS.2020.33293.1137
[41] Foscarini, F., Bellocchi, G., Confalonieri, R., Savini, C. and VandenEede, G. (2010) . "Sensitivity analysis in fuzzy systems. " Integration of SimLab and DANA. Environmental Modelling & Software, (2010):1256-1260. https://doi.org/10.1016/j.envsoft.2010.03.024
[42] SimLab ver.3.2.6. "Development Framework for Uncertainty and Sensitivity Analysis. " Joint Research Centre of the European Commission. Econometric sand Applied Statistics, Ispra, Italy.http://simlab.jrc.ec.europa.eu. (2009).
[43] Ekström P.A. (2005). "Simulation Toolbox for Sensitivity Analysis". Degree Project
[44] Conover, W. J. "Practical nonparametric statistics. " John Wiley & Sons, New York. (1980).
[45] Hamby, D. M. (1994). "A review of techniques for parameter sensitivity analysis of environmental models. " Environmental monitoring and assessment, 32.2 (1994): 135-154, . https://doi.org/10.1007/BF00547132.