



# Comparative study of numerical methods used in prediction of post-crack behavior of waffle slabs

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## Abstract

One effective way to reduce dead and earthquake loads is to use waffle slabs. The use of waffle slabs leads to smaller sections. There are various methods for predicting the behavior of slabs, the most important of which is fracture mechanics. Considering the variety of geometry of waffle slabs, it is important to investigate the formation and growth of cracks in them. The current research aims to analyze the post-crack behavior of waffle slabs under common loading conditions using the finite element method by using numerical modeling to analyze cracks in reinforced concrete members. by comparing the obtained results with the laboratory sample, the optimal numerical model for predicting the behavior of the hollow slab was determined. The waffle slab sample was modeled with two types of distributed and concentrated loading conditions and two types of simple and fixed support conditions. The models were modeled by two methods of concrete damage plasticity and smeared cracked concrete by Abaqus software. Various parameters of these samples, including the load-displacement diagram, the load coincident with the formation of cracks, the displacement in the middle of the slab opening, and the pattern of the formation and propagation of cracks, etc., were discussed and investigated. To validate and control the modeling process, the laboratory results of the research of Nithyambigai G et al. in 2021 have been used. The outcomes of this study showed; a relatively good correspondence between the two methods of Concrete damage plasticity and the concrete smeared crack model, but due to the definition of failure in the Concrete damage plasticity method, more accurate results can be obtained. © 2017 Journals-Researchers. All rights reserved (DOI:<https://doi.org/10.52547/JCER.4.4.12>)

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## 1. Introduction

The most significant challenge for engineers in analyzing the behavior of reinforced concrete members is to predict how cracks will form and

propagate in them. Also, the effect of the formation of cracks on various components such as hardness, load, deformations, and durability in reinforced concrete members is important. one of the important components of structures that are responsible for carrying a major part of the load on the structure is slabs. Methods such as finite element analysis are used to predict the formation and expansion of cracks

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in reinforced concrete members like slabs. Abaqus is one of the software for finite element modeling and analysis. This software can simulate reinforced concrete members.

Classical models for concrete cracking are based either on the so-called smeared crack approach or the discrete crack approach. Smeared crack models are based on the theory of continuum mechanics and are characterized by spreading the dissipated energy along the width of the localization band. The class of discrete crack models is characterized by incorporating the discontinuity of the displacement field due to cracking directly into the finite element formulation to capture the strong discontinuity kinematics of a discrete crack.[1] In the early days of finite elements, analysis cracks were modeled using separation between element edges (Ngo and Scordelis 1967, Nilson 1968). The approach suffers from two drawbacks. First, it implies a continuous change in nodal connectivity, which does not fit the nature of the finite element displacement method. Secondly, the crack is constrained to follow a predefined path along the element edges. The counterpart of the discrete crack concept is the smeared crack concept, in which a cracked solid is imagined to be a continuum. The approach introduced by Rashid (1968) starts from the notion of stress and strain and permits a description in terms of the stress-strain relation. This approach preserves the topology of the original finite element mesh and does not impose restrictions concerning the orientation of crack planes.[2]

Bangash (1989) has provided a step-by-step definition for modeling cracks from their formation and propagation to opening and closing under stepwise loading. In this definition, where the cracks are formed, the tensile stress cannot be directly defined in the direction perpendicular to the cracks. A concrete element parallel to the crack is considered to receive stresses defined based on biaxial or triaxial continuity relations in the plane parallel to the crack.[3]

Three methods of concrete damage plasticity, concrete smeared cracking and brittle cracking are used for crack modeling in Abaqus software. The generalized concrete damage plasticity model is the Drucker-Prager failure criterion, and it is one of the strong theories in reinforced concrete failure

modeling. The concrete damaged plasticity model assumes nonassociated potential plastic flow. The flow potential used for this model is the Drucker-Prager hyperbolic function. Drucker Prager's (1952) rupture criterion is a generalized Mohr-Coulomb rupture criterion for all types of soil.[4] The model makes use of the yield function of Lubliner et. al. (1989), with the modifications proposed by Lee and Fenves (1998) to account for different evolution of strength under tension and compression.[5][6] in the concrete smeared cracking method, when there is no reinforcement in significant regions of a concrete model, the strain softening approach for defining tension stiffening may introduce unreasonable mesh sensitivity into the results. Crisfield (1986) discusses this issue and concludes that Hiller Borg's (1976) proposal is adequate to allay the concern for many practical purposes. Hiller Borg defines the energy required to open a unit area of crack as a material parameter, using brittle fracture concepts.[7] With this approach, the concrete's brittle behavior is characterized by a stress-displacement response rather than a stress-strain response. Under tension, a concrete specimen will crack across some sections. After it has been pulled apart sufficiently for most of the stress to be removed (so that the elastic strain is small), its length will be determined primarily by the opening at the crack.

## 2. Methodology

This research has been done with the ABAQUS finite element software. The background of the work and relevant theories in the field of crack analysis in reinforced concrete sections were studied. The results of valid laboratory studies in the field of waffle slabs were selected and the relevant samples were modeled and analyzed. Using the results obtained from the numerical analysis, the pattern of crack formation and propagation and load-displacement diagram were compared with the laboratory samples. Software analysis is given in two parts. The first part is about validation. To validate the results obtained from Abaqus, they were compared with the laboratory samples of the article (Behavior of waffle slab Nithyambigai G (2021)).[8] In the second part, the crack formation and propagation pattern and the load-displacement diagram of numerical samples under

different loading and support conditions were compared with concrete damage plasticity and concrete smeared cracking methods.

2.1. validation

Five samples of waffle slabs have been investigated in the reference article under the title of hollow slab behavior. The slabs have different

dimensions on a scale of 1 to 5. Samples of waffle slabs have been compared with samples of flat slabs. Concrete with a compressive strength of 47 MPa and a modulus of elasticity of 31877 MPa has been used. Rebars with a yield strength of 400 MPa and ultimate strength of 600 MPa and modulus of elasticity of 200000 MPa were used. Table (1) shows the mentioned values.

Table 1

Concrete mechanical properties

Material type	Compressive strength (MPa)	modulus of elasticity (MPa)	yield strength (MPa)	ultimate strength (MPa)
concrete	47	31877	-	-
rebar	$2 \times 10^5$	-	400	600

To simulate the compressive stress behavior of concrete in the CDP method, the modified Hogenstad (1951) relation has been used.[9] Equations (1) and (2) represent the modified Hognestad.

$$f_c = f'_c \left[ 2 \left( \frac{\epsilon_c}{\epsilon_{cu}} \right) - \left( \frac{\epsilon_c}{\epsilon_{cu}} \right)^2 \right] \quad \text{for } 0 \leq \epsilon_c < \epsilon_c \quad (1)$$

$$f_c = f'_c [1 - Z(\epsilon_c - \epsilon_{cu})] \quad \text{for } \epsilon_0 \leq \epsilon_c < \epsilon_{cu} \quad (2)$$

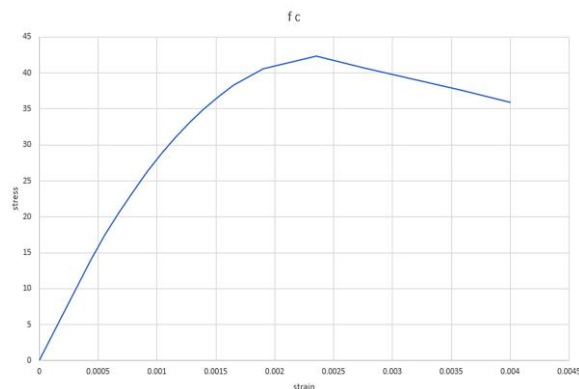


Fig. 1. A Uniaxial compressive stress-strain diagram of concrete

In the upcoming research, the modified model of Wahalathantri Buddhi et.al, (2011) has been

used.[10] This model is the modified method of Nayal and Rashid. [11]

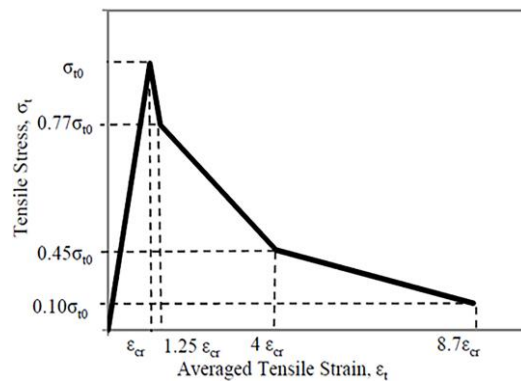


Fig. 2. Wahalathantri tension stiffening model

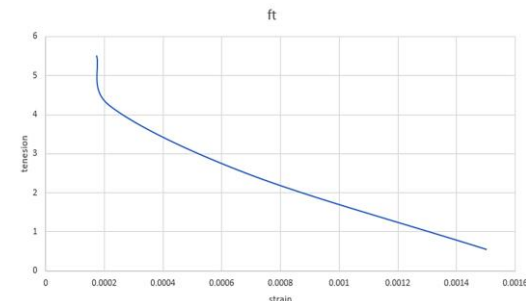


Fig. 3. Uniaxial tensile stress-strain diagram of the sample

The compressive and tensile failure parameter in the CDP method is defined by equation (3).

$$d_c = 1 - \frac{\sigma_c}{f_c} \quad , \quad d_t = 1 - \frac{\sigma_t}{f_t} \quad (3)$$

For the smeared cracking method, the pressure values are the same as the plastic failure. For the concrete-smeared cracking method, the tension stiffening effect must be estimated; it depends on such factors as the density of reinforcement, the quality of the bond between the rebar and the concrete, the relative size of the concrete aggregate compared to the rebar diameter, and the mesh. when there is no reinforcement in significant regions of a concrete model, the strain softening approach for defining tension stiffening may introduce unreasonable mesh sensitivity into the results. Crisfield (1986) discusses this issue and concludes that Hillerborg's (1976) proposal is adequate to allay the concern for many practical purposes. Hillerborg defines the energy required to open a unit area of crack as a material parameter, using brittle fracture concepts. With this approach, the concrete's brittle behavior is characterized by a stress-displacement response rather than a stress-strain response. For linear elements, the amount of displacement is obtained by multiplying the length of the element by the amount of strain coinciding with the crack initiation, which is equal to  $5 \times 10^{-5}$ .

## 2.2. Modeling

Abaqus finite element analysis software was used for the modeling and analysis of samples. Fig. 4. Shows a waffle slab modeled in Abaqus.

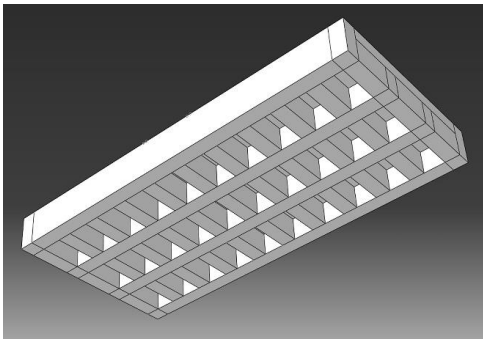


Fig. 4. Schematic view of waffle slab

The waffle slab was modeled with a scale of 1:5 and dimensions of 90 x 500 x 1020 mm. The beam sizes are 60x40mm and were tied in various spacing. Beams were tied in an interlocking manner as a grid. The reinforcement detailing on the beam in top 1# 6mmØ and bottom 1# 6mmØ. Stirrups for the beam are 30 mm \_ 50 mm of 6 mm Ø. The stirrups were placed at a distance of various c-c. The top slab is 30 mm thick and the reinforcing details of 100 mm spacing of 8 mm Ø.

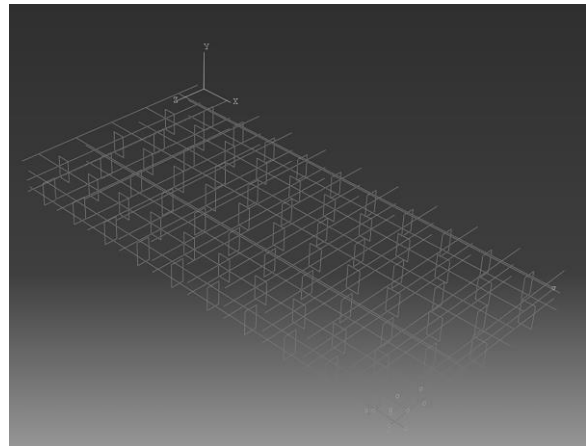


Fig. 5. Schematic view of rebars

The experimental sample has simple supports and the load is concentrated on a 130x130 mm plate. The results of the load- displacement of concrete damage plasticity and concrete smeared cracking are shown in figures (6) and (7).

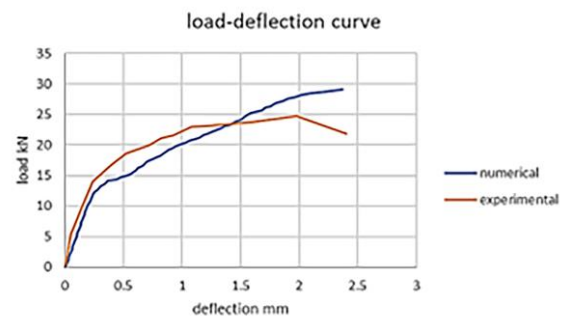


Fig. 6. Comparison of displacement force diagram of concrete damage plasticity model with the experimental sample in concentrated load mode with simple support

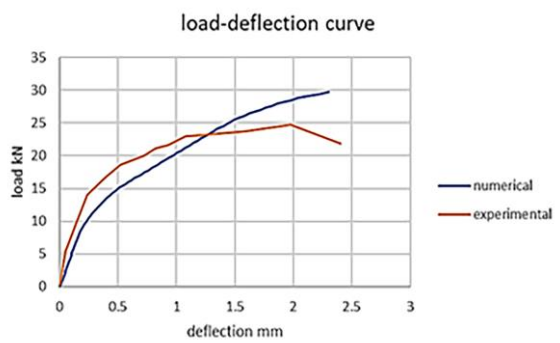


Fig. 7. Comparison of displacement force diagram of concrete smeared cracking model with the experimental sample in concentrated load mode with simple support

By comparing the load-displacement diagram of the experimental and numerical samples, it was determined that there is a good match between them.

### 2.3. Boundary condition

Two types of boundary conditions were modeled. One is a simple support and the other is a fixed support. Fig.8. and Fig.9.



Fig. 8. Simple support condition



Fig. 9. fixed support condition

### 2.4. loading

Four types of boundary conditions and different loading are modeled. waffle slab with simple support and concentrated loading, waffle slab with simple support and distributed loading, waffle slab with fixed support and concentrated loading, and waffle slab with fixed support and distributed loading.

In the Specimens with concentrated load, the load is applied as a pressure on a part of the middle of the slab with dimensions of 130x130 square mm. In specimens with a uniform load, the load is applied widely on the upper part of the slab.

## 3. Results and discussion

The results of the load-displacement diagram of two concrete damage plasticity and concrete smeared cracking methods for the four named specimens are shown in Fig.10. to Fig.13.

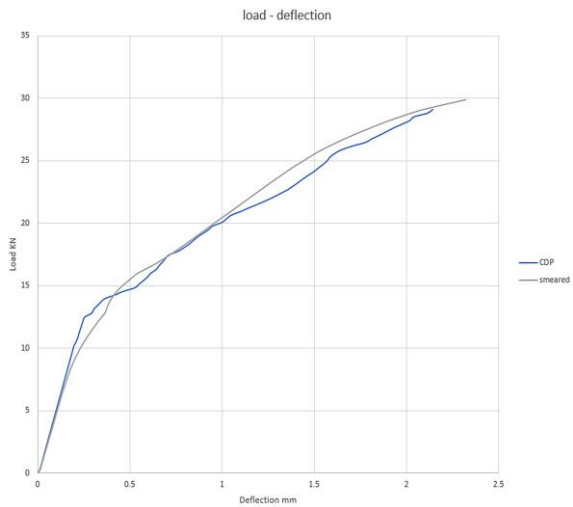


Fig. 10. Comparison of load-displacement of two concrete damage plasticity and concrete smeared cracking methods in case of concentrated load with simple support

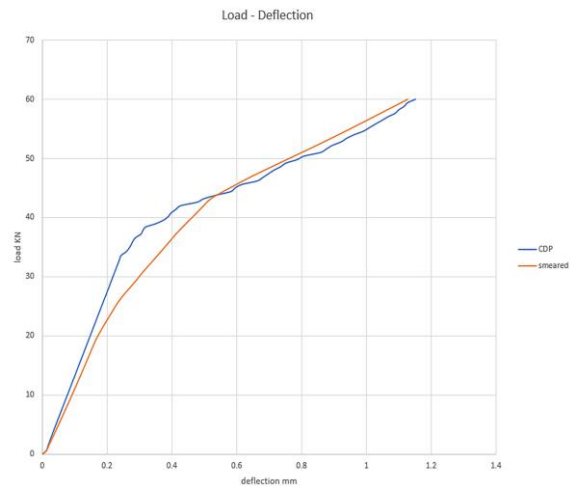


Fig. 11. Comparison of load-displacement of two concrete damage plasticity and concrete smeared cracking methods in case of uniform load with simple support

In the simple supported concentrated load case, both methods are similar up to a loading of 7 KN, but after that, the concrete smeared cracking method is almost trilinear. For the case of uniform load with simple support, the incline of the linear part of the diagram of the concrete damage plasticity model is higher and this linear function continues up to the load of 33.6 KN. However, in the concrete smeared cracking method, the performance is still almost

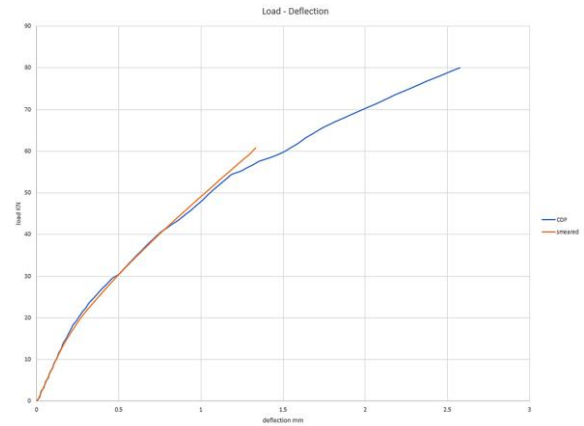


Fig. 12. Comparison of load-displacement of two concrete damage plasticity and concrete smeared cracking methods in case of concentrated load with fixed support

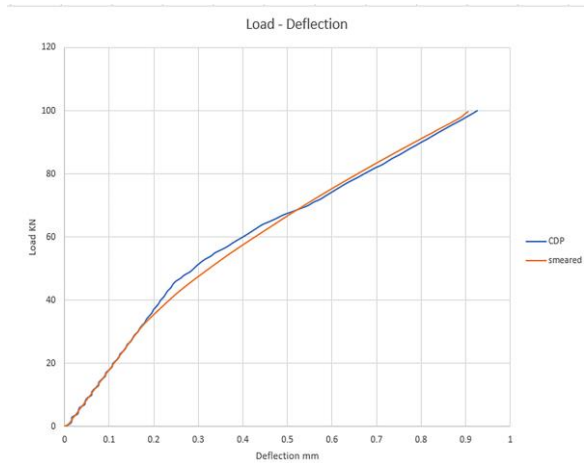


Fig. 13. Comparison of load-displacement of two concrete damage plasticity and concrete smeared cracking methods in case of uniform load with fixed support

three-line, and the incline of the initial part is gentler and has a linear performance up to a load of 19.2 KN. In the case of a concentrated load with fixed support, the two methods are in good agreement with each other, and the incline of the graphs is almost the same. In the concrete damage plasticity method, the loading is linear up to the load value of 18 KN and this value is equal to 15 KN for the concrete smeared cracking method. Due to the problems of convergence in the concrete smeared cracking

method, it was calculated up to 60 KN. For the case of uniform load with fixed supports, the slope of the linear part of the graph of the CDP method is higher and this linear function continues up to a load of 45 KN. The concrete smeared cracking method is still almost three-linear. The incline of the initial part is almost the same as the concrete damage plasticity method and has a linear performance up to a load of 33 KN. In general, the displacement load values of the two methods are in good agreement with each other for all specimens.

For the concrete damage plasticity method, tensile damage is used as a criterion to identify the first crack. The maximum principal plastic strain criterion is used to identify the first crack for the concrete smeared cracking method. The chart comparing the initiation crack load for both methods is given in Fig.14. and Fig.15.

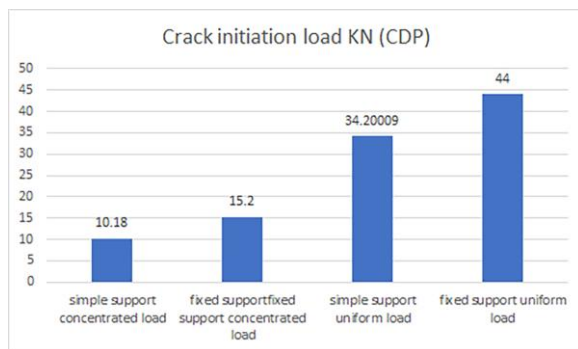


Fig. 14. Initial cracking load for concrete damage plasticity

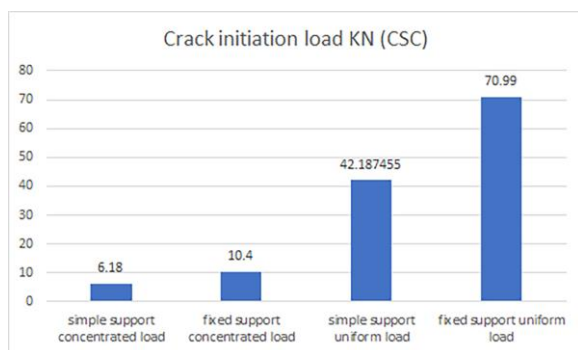


Fig. 15. Initial cracking load for concrete smeared cracking

#### 4. Conclusions

The following points were concluded based on the values obtained from the model.

- The concrete smeared cracking method can only be used for static analysis, and the analysis process based on time increments in this method cannot continue until the end, and eventually, too many attempts made for this increment will be shown. In the concrete smeared cracking method, some specimens did not complete the analysis until the end, which caused less accuracy in the obtained answer.
- The initial crack load for waffle slab with simple support and concentrated loading, waffle slab with fixed support and concentrated loading, waffle slab with simple support and uniform loading, and waffle slab with fixed support and distributed loading, respectively, for concrete damage plasticity model 10.18, 15.2, 34.2, 44 KN has been obtained. And for concrete smeared cracking model 6.18, 10.4, 42.19, and 70.99 KN has been obtained.
- The reason for the huge difference in the initial cracking load in the specimen of fixed support with uniform load is the lack of definition of tensile damage in the concrete smeared cracking method. Because the initial crack in the concrete damage plasticity method was determined using the tensile damage criterion at a load of 44 KN, while in the concrete smeared cracking method, the crack location is determined only through the maximum principal plastic strain.
- The load-displacement diagram of the two methods is almost identical in the linear region, but in the non-linear region, the concrete smeared cracking method is almost trilinear.
- The compatibility of the load-displacement diagram of the two methods is more in the case of fixed support.

#### References

- [1] Hofstetter Guenter, Meschke Gunther, 2011, Numerical Modeling of Concrete Cracking, Springer Science & Business Media.

- [2] Rots, J.G., Blaauwendraad, J., 1989, Crack Models for Concrete, Discrete or Smearred.Fixed, Multi-Directional or Rotating., Delft University of Technology.
- [3] Bangash. M. Y. H., 1989, Concrete and Concrete structures: Numerical modeling and applications, Elsevier.
- [4] Drucker DC, Prager W, (1952), Soil mechanics and plastic analysis or limit design. Q Appl Math, pp 157–165
- [5] Lubliner, J., J. Oliver, S. Oller, and E. Oñate, “A Plastic-Damage Model for Concrete,” *International Journal of Solids and Structures*, vol. 25, pp. 299–329, 1989.
- [6] Lee, J., and G. L. Fenves, “Plastic-Damage Model for Cyclic Loading of Concrete Structures,” *Journal of Engineering Mechanics*, vol. 124, no.8, pp. 892–900, 1998.
- [7] Hillerborg, A., M. Modeer, and P. E. Petersson, “Analysis of Crack Formation and Crack Growth in Concrete by Means of Fracture Mechanics and Finite Elements,” *Cement and Concrete Research*, vol. 6, pp. 773–782, 1976.
- [8] Niyhyambigai G., Rameshwaran P.M., Stella Mary F., (2021), “Behavior of waffle slab”, Elsevier, Vol 46, No 9, pp 3765-3768
- [9] Hognestad E., (1951), “A Study of Combined Bending and Axial Load in Reinforced ConcreteMembers”, Bulletin 399, University of Illinois Engineering Experiment Station Urbana, pp 128.
- [10] Wahalathantri Buddhi. L., Thambiratnam David, Chan Tommy, Fawzia Sabrina, (2011), “A Material Model for Flexural Crack Simulation in Reinforced Concrete Elements Using ABAQUS”, First International Conference on Engineering, Designing and Developing the Built Environment for Sustainable Wellbeing, Queensland University of Technology, Australia, pp 260-264
- [11] Nayal, R., Rasheed, H.A. (2006). Tension Stiffening Model for Concrete Beams Reinforced with Steel and FRP Bars. *Journal of Materials in Civil Engineering*, Vol 18, No 6, pp 831-841.