J-Researchers

Journal of Civil Engineering Researchers

Journal homepage: www.journals-researchers.com

# Investigating the Effect of Uncertainty in the Optimal Design of a Trapezoidal Channel

S. Pourbakhshian,[a*] F. Abdolrasuli,[a]

[a] *Department of Civil Engineering, Ramsar Branch, Islamic Azad University, Ramsar, Iran*

**ABSTRACT**

In this article, the optimal shape of the compound trapezoidal cross-section is presented by considering the deterministic constraints and the probabilistic constraint of channel flooding as uncertainty with the SPSA algorithm and with three flow discharge of 10, 50, and 120 m3/s. The objective function is to minimize the cost of excavation and lining, the design variables of depth and bed width of the canal, side slopes and constraints include uniform flow, maximum and minimum velocity, water surface width and the overtopping probability. The values of the freeboard and the slope of the channel bed are fixed and equal to 0.5 meters and 0.0028, respectively. The results show that with the increase in overtopping probability, the flow depth increases, but the side slopes, velocity and Froud number decrease. The cost of construction and the bed width, initially decreases with the increase of the overtopping probability, and at a certain value, the probability reaches its minimum value, and then with the increase of the overtopping probability, the cost of construction and bed width increases.



**ARTICLE INFO**

Received: October 17, 2024
Accepted: November 26, 2024

***Keywords:***
*Open channels*
*Optimal design*
*SPSA algorithm*
*Overtopping probability*
*Uncertainty*

DOI: 10.61186/JCER.6.4. 40
DOR: 20.1001.1.22516530.1399.11.4.1.1

## 1. Introduction

The optimal design of the cross section of the channel is necessary for the design discharge to convey through it and the least cost for its construction [1]. The optimal design of the composite channel cross-section was first addressed by Trout (1982) with the aim of minimizing lining costs with composite roughness for trapezoidal, rectangular, and triangular channels [2] .Guo, and Hughes, (1984) for the first time considered the free board as a design variable for the optimal design of the trapezoidal channel using the first principles of calculus and derivation and with the objective function of frictional resistance or the construction cost [3]. In Monadjemi's 's research (1994), the triangular section is the best hydraulic section among different sections such as rectangular, triangular, trapezoidal and round bottom triangular [4]. Swamee (1995) and Swamee et al. (2001, 2002, 2002) Presented optimal channels with triangular, rectangular, trapezoidal, and circular sections, considering water lost as seepage and evaporation losses, and using presented a non-linear method. The results showed that the cross-sectional area of the channel and losses due to seepage and evaporation in a trapezoidal channel are less than triangular and rectangular channels. The results of Babaeyan-Koopaei et al.'s research (2000) showed that the cross-sectional the wetted flow area and wetted perimeter

* Corresponding author. Tel.: +98-9113967847; e-mail: Sommayyeh. pourbakhshian@gmail.com.

of a triangular channel with a parabolic bottom is less than a channel with a triangular and parabolic cross-section. Das (2000) presented the optimal shape of the cross-section with composite roughness trapezoidal channel with freeboard using the method of Lagrange coefficients, taking into account the constraints of Manning's equation and the objective function of the construction cost [12]. Jain et al. (2004) presented the optimal design of the trapezoidal channel section by considering the velocity and top width constraints of the water surface using genetic algorithm. The cost of construction a channel using genetics algorithms is lower compared to the method of Lagrange multiplier. Bhattacharjya (2005) considered problem for optimal design of composite channel cross section of trapezoidal shape by investigating the constraint of freeboard and changes in the specific energy of the channel. [14]. Bhattacharjya (2006) optimized the trapezoidal channel section with nonlinear optimization model incorporating the critical flow condition of the channel. The results show that this method has a better and more favorable performance compared to the Lagrange method in the research of Das. [15]. Also, Bhattacharjya used the optimal shape of the open channel cross-section with the combined method of geneti algorithm and second-order sequential programming algorithm. The results showed the high efficiency of the mentioned method in the optimal and stable design of open channels. [16]. Das (2007) presented the optimal cross-section model of a trapezoidal channel with two objective functions of minimizing the overall cost of channel construction and minimizing the probability of overtopping and limiting the establishment of uniform flow using the first-order analysis method. The results of this research showed that with the decrease of overtopping probability, the flow depth decreases and the bed width increases. [17]. Bhattacharjya and Satish (2008) developed the model of Das (2007) by considering the freeboard as a design variable and also using genetic algorithm. The results indicate a direct relationship between the width of the channel and the cost of its construction, as well as the better efficiency of the genetic algorithm compared to the classical optimization. [18]. Reddy and Adarsh (2010) presented the optimal shape of the cross-section of the composite trapezoidal channel using the elitist-mutated particle swarm optimization (EMPSO) method with overtopping constrained design. The results show that the cost of construction the channel is lower than the method of Lagrange coefficients. [19]. While in the research of Das (2010), the mentioned clause was obtained through risk analysis. [20]. In another study, Reddy and Adarsh (2010) used two meta-heuristic methods such as Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) algorithm to obtain the optimal shape of the channel section [21]. In a comprehensive research, Adarsh (2012) designed the optimal shape of the trapezoidal channel by taking into account the water loss in the form of seepage and evaporation using the meta-heuristic optimization technique namely probabilistic global search Lausanne (PGSL) [23]. Adarsh and Reddy (2013) presents a probabilistic multi-objective model for optimal design of composite channels that have across-sectional shape of horizontal bottom and parabolic sides with three objective functions including minimizing the cost of channel construction, maximizing the probability of the expected the channel capacity being greater than the design discharge and minimizing the overtopping using the particle swarm optimization method and Pareto-optimal. The results showed that the presented approach has good potential for other sections designs of open channels under input parameter uncertainty. In recent years, Easa (2011), Easa and Vatankhah, (2014), Han (2015), Han and Easa (2017), very extensive researches regarding the design of new general open channel cross-sections were introduced. The results show that the optimal section that minimizes construction cost is substantially better than the most hydraulically efficient section. [24-27]. Orouji et al. (2016) optimized the trapezoidal compound channel section using the frog leaping algorithm. The results show that the use of this algorithm is more economical compared to genetic algorithms, ants, etc. [28]. Roushangar et al. (2018) in their studies to provide the optimal shape of the composed trapezoidal channel using genetic algorithm showed that the application of depth, speed and Froud number constraints increases the construction cost, while the water surface width constraint reduces the cost. . [29]. Gupta et al. (2018) used Fish Shoal Optimization (FSO) for the optimal design of the trapezoidal channel section. which reduced the cost of channel construction compared to particle swarm algorithm (PSO) [30]. Farzin and Anarki (2020) using meta-heuristic methods, to the optimal and probabilistic design of the composed trapezoidal channel cross-section in which models are based on constant or variable freeboard, uniform or composite roughness coefficient, fixed and variable freeboard, and also velocity, Froude number and overtopping constraints using bat hybrid algorithm. The results showed that using HBP, compared to BA, PSO, LINGO, Lagrange multiplier method and shuffled frog-leaping algorithm, led to a 32% reduction in the cost of channel construction. Therefore, HBP has high potential for the optimal design of open channels. [31]. Pourbakhshian and Pouraminian (2021) presented some analytical models for the optimal design of trapezoidal composite channel cross-section. The objective function is the cost function per unit length of the channel, which includes the excavation and lining costs. To define the system, design variables including channel depth, channel width, side slopes, freeboard, and roughness coefficients were used. The constraints include the mannin-

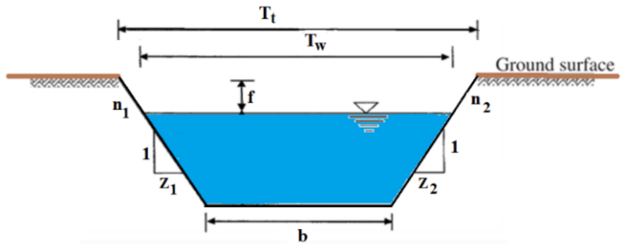


Figure 1. Representation of the cross-sectional geometry of the composite trapezoidal channel

Mg's equation, flow velocity, Froude number and water surface width, The Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm was used to solve the optimization problem. The results are presented in three parts. In the first part, the optimal values of the design variables and the objective function are presented in different discharges. In the second part, the relationship between cost and design variables in different discharges is presented in the form of conceptual and analytical models and mathematical functions. Finally, in the third part, the changes in the design variables and cost function are presented as a graph based on the discharge variations. Result indicates that the cost increases with increasing water depth, left side slope, equivalent roughness coefficient, and freeboard. [32].

the optimal cross section of a composite parabolic channel by considering four models based on freeboard changes. The results show that increasing the discharge increases the flow depth, left and right-side slopes, total top width and water surface width, channel cross-sectional area and flow area, the total channel perimeter and wetted perimeters, flow velocity, Froud number and the cost increases. By examining the relationship between cost with design variables, constraints and geometric parameters of parabolic channel cross section in different iterations, the characteristic of the model that won the most number of iterations is based on the increase of left and right side slopes, total top width and water surface width, the cross-sectional area of the canal and the flow area, the total channel perimeter and the wetted perimeters, the Froud number, increase the cost, and in contrast, increasing the depth and flow velocity reduces the cost. Comparing the results of the four models with each other shows that the cost values in the first model are less than other models and in the third model are higher than all models. [33]. Since parameters such as discharge, longitudinal bed slope of the channel and roughness coefficients of the property have a stochastic nature, so that with the passage of time, the longitudinal bed slope of the channel and roughness coefficients will undergo changes due to the phenomenon of erosion and sedimentation compared to the initial values for the design and the amount Due to the occurrence of unauthorized rainfall, the discharge through the channel is higher than the design discharge of the channel, so there is uncertainty in determining the depth of the flow in the channel and the overtopping of the channel. As a result, considering the optimal design of irrigation channels limiting the possibility of overtopping reduces costs and damages. [34].

In this research, the optimal design of the cross-section of the composite trapezoidal channel by considering the deterministic constraints and the overtopping constraint and uncertainty by using the optimization algorithm of the Simultaneous Perturbation Stochastic Approximation (SPSA) in the flow discharge of 10, 50 and120 (m^3/s). Since it is necessary to design the optimal cross-section of open channels with the aim of passing the maximum flow and the minimum construction cost, in this regard, it is of great importance in terms of the possibility of overtopping due to the uncertainty of the design parameters including flow rate, longitudinal bed slope and flow roughness coefficients. In this research, for the first time, the SPSA algorithm has been used for the optimal design of composite trapezoidal channels by considering overtopping and uncertainty, and in this regard, the most important part of innovation is to include a complete set of definite constraints and the possibility of overtopping in the values of different flows.

## 2. Optimal design of open channels

### *2.1. Trapezoidal cross section*

As shown in Fig. 1, $z_1:1, z_2:1$represent the side slopes of the channel corresponding to the left and right sides, $n_1, n_2, n_3$ are Manning's roughness coefficients on the left, right and bottom sides of the channel, b is bottom width, $y$ is the flow depth, $f$ is the freeboard, and $s_0$ is the longitudinal bed slope. where $A_t$ and $P_t$ are the is the total channel cross-sectional area and perimeter, respectively; $T_t$is the total top width of the channel cross-section; $A_w$and $P_w$are the channel wetted area and perimeter, respectively; $T_w$is the water surface width, $P_{w1}$, $P_{w2}$, and $P_{w3}$ are wetted perimeters corresponding to the left, right and bottom sides of the channel respectively. $P_{t1}$, $P_{t2}$, and $P_{t3}$are perimeters corresponding to the left, right and bottom sides of the channel, respectively; $R_w$ is the hydraulic radius; and $D$is the hydraulic depth. Parameters $y$, $b$, z1, z2, and f are defined in Fig. 1 are related to the area (Eq 1), perimeter (Eq 2), and the overall width of the section (Eq 3):

$$A_t = b(y+f) + (z_1+z_2)\,(y+f)^2/2 \tag{1}$$

$$P_{t1} = ({z_1}^2+1)^{(1/2)}.(y+f)$$

$$P_{t2} = ({z_2}^2+1)^{(1/2)}.(y+f)$$

$$P_{t3} = b$$

$$P_t = P_{t1}+P_{t2}+P_{t3} \tag{2}$$

$$P_t = \left\{\left[({z_1}^2+1)^{(1/2)} + ({z_2}^2+1)^{(1/2)}\right](y+f) + b\right\}$$

$$T_t = b + (z_1+z_2)(y+f) \tag{3}$$

In the equations 1 to 3, by removing the freeboard in the equations, the corresponding water flow equations are obtained.

### *2.2. Design variable*

According to the geometric model described in figure (1), design variables, description and their minimum and maximum values are given in Table 1.

Table 1.
Representation of design variables

| $X_i$ | unit | Design variables | $X_L$ | $X_R$ |
|---|---|---|---|---|
| $x_1 = y$ | m | Depth of flow | 0.5 | 15 |
| $x_2 = b$ | m | Channel bed width | 1 | 20 |
| $x_3 = z_1$ | - | Channel right side slope | 0.2 | 3 |
| $x_4 = z_2$ | - | Channel left side slope | 0.2 | 3 |

### *2.3. Constraint*

#### *2.3.1. Deterministic constraint*

According to Table 2, the constraint of uniform flow is considered to conduct the uniform flow in the channel [6]. Therefore, in this research, the Manning's equation constraint Horton's method was used to control uniform flow and to calculate the equivalent roughness coefficient [35]. Froud number constraint is included in order to avoid development of critical flow in the optimal design of the channel [14]. And the Channel top width is included to control the cost of land acquisition [12, 13, 21]. The minimum allowed velocity to prevent sedimentation is in the range of 0.6 to 0.9 m/s and the minimum velocity to prevent the growth of vegetation is 0.75 m/s [1]. The minimum velocity allowed in channel design is in the range of 0.75 to 0.9 [6]. In channels with a rigid boundary, the maximum allowable velocity (VL) is the velocity that does not cause erosion. Moreover, to ensure the conveyance of the discharge through the cross-section, the mean actual flow velocity in the channel should not exceed the maximum permissible velocity [8,13]. In this study, the minimum and maximum velocity values are 0.75 and 4 m/s, respectively.

Table 2
Representation of design variables

| Equation | Describe | Constraint |
|---|---|---|
| $\Phi_1 = \left\lvert \frac{Q}{\sqrt{S_0}} - \frac{A_w^{5/3}}{\sum_{i=1}^{2} n_i P_{wi}^{1.5}} \right\rvert - \varepsilon \le 0$ | Uniform flow | $\boldsymbol{\Phi_1}$ |
| $\Phi_2 = \frac{F_r}{F_{r_{max}}}$ | Sub Critical flow | $\boldsymbol{\Phi_2}$ |
| $\Phi_3 = \frac{T_t}{T_{max}} - 1 \le 0$ | Channel top width | $\boldsymbol{\Phi_3}$ |
| $\Phi_4 = \frac{V_{min}}{V_{ave}} - 1 \le 0$ | Minimum permissible velocity | $\boldsymbol{\Phi_4}$ |
| $\Phi_5 = \frac{V_{ave}}{V_{max}} - 1 \le 0$ | Maximum permissible velocity | $\boldsymbol{\Phi_5}$ |

#### *2.3.2. Overtopping probability constrained*

The overtopping probability constrained for the optimal design concept is used to design for safety against overtopping [20].

The phenomenon of erosion and sedimentation as well as runoff caused by unexpected rainfall can cause changes in the design values of longitudinal slope, channel roughness coefficients and flow discharge, which brings uncertainty in the depth of flow in the channel. Therefore, in this regard, from the first order analysis of uncertainty and assuming a normal distribution for the design parameters and the amount of flow passing through the channel due to the occurrence of precipitation, there may be changes compared to the initial design values that lead to uncertainty. The condition of the probability of overtopping is introduced as follows: The condition of the probability of overtopping is in the form of Eq. 4.

$$\Phi_6 = p(y > y+f) = P \tag{4}$$

$p(y > y+f)$, It is the probability of exceedance of flow depth over the freeboard which should be equal to the constant value of p.

The overtopping probability constraint is obtained in the order of the following steps:

1) Determination equation of Horton's equation in Manning's equation:

$$Q = \sqrt{S_0}\,\frac{A_w^{5/3}}{\left(\sum_{i=1}^{3} n_i P_{wi}^{1.5}\right)^{\frac{2}{3}}} \tag{5}$$

2) Due to the probabilistic nature of flow discharge, bed slope and channel roughness coefficients, it is derived from the Eq. 6 to 8 with respect to parameters $y, n_3, n_2, n_1, S_0$:

$$\frac{dQ}{dy} = Q[\frac{5}{3}\frac{1}{A_w}\frac{dA_w}{dy} - \frac{2}{3}\frac{A_w^{5/3}}{(\sum_{i=1}^{3} n_i^{3/2}P_{wi})}\frac{d}{dy}((\sum_{i=1}^{3} n_i^{3/2}P_{wi})] \quad (6)$$

$$\frac{dQ}{dn_i} = -Q[\frac{2}{3}\frac{1}{(\sum_{i=1}^{3} n_i^{3/2}P_{wi})}\frac{d}{dn_i}((\sum_{i=1}^{3} n_i^{3/2}P_{wi})] \quad (7)$$

$$\frac{dQ}{dS_o} = \frac{Q}{2S_o} \quad (8)$$

3) According to Chow's (1988) research, the variance of y ($S^2{}_y$) is dependent on the variance of Q ($S^2{}_Q$), variance of $n_1$ ($S^2{}_{n_1}$), variance of $n_2$ ($S^2{}_{n_2}$), variance of $n_3$ ($S^2{}_{n_3}$) and variance of $S_o$ ($S^2{}_{S_o}$).

$$S^2{}_y = \frac{1}{(\frac{dQ}{dy})^2}[S^2{}_Q + (\frac{dQ}{dn_1})^2 S^2{}_{n_1} + (\frac{dQ}{dn_2})^2 S^2{}_{n_2} + (\frac{dQ}{dn_3})^2 S^2{}_{n_3} + (\frac{dQ}{dS_o})^2 S^2{}_{S_o} \quad (9)$$

By arranging equation (9) based on $dQ/dy$, the Eq. (9) is written as Eq. (10):

$$\frac{dQ}{dy} = \frac{1}{S_y}[S^2{}_Q + (\frac{dQ}{dn_1})^2 S^2{}_{n_1} + (\frac{dQ}{dn_2})^2 S^2{}_{n_2} + (\frac{dQ}{dn_3})^2 S^2{}_{n_3} + (\frac{dQ}{dS_o})^2 S^2{}_{S_o}]^{\frac{1}{2}} \quad (10)$$

4) Therefore, by substituting Eq. (6), (7) and (8) in equation (10), the standard deviation of the flow depth is obtained according to the following equation:

$$S_y = \frac{3[\frac{S^2{}_Q}{Q^2} + \frac{1}{(\sum_{i=1}^{3} n_i^{3/2}P_{wi})^2}\sum n_i P_{wi}{}^2 S^2{}_{n_i} + \frac{1}{4S_o{}^2}S^2{}_{S_o}]^{\frac{1}{2}}}{\frac{5}{A}\frac{dA_w}{dy} - \frac{2}{(\sum_{i=1}^{3} n_i^{3/2}P_{wi})}\frac{d}{dy}((\sum_{i=1}^{3} n_i^{3/2}P_{wi})]} \quad (11)$$

5) Determining the standard deviation of the flow depth based on the normal standard variable.
The standard normal variable Z is defined as follows [6]:

$$Z = \frac{X-\mu}{\sigma} \quad (12$$

In statistical calculations, $\mu = \bar{X}$ $\sigma = S_x$ and $Z$is the standard normal distribution variable whose value is determined according to the overtopping probability values and interpolation in the cumulative distribution table of normal distribution probabilities. In this research, (y) is replaced for the variable. $(X-\mu)$, Therefore, the variable is defined as follows:

$$Z = \frac{y+f-y}{S_y} = \frac{f}{S_y} \quad (13)$$

In equation (13), $f$ it is the freeboard of the channel, which in this research is equal to 0.5meters.

$$S_y = \frac{f}{Z} \quad (14)$$

By setting Eq. (14) and Eq. (11) equal, the overtopping constraint is defined as the following equation.

$$\frac{f}{Z} = \frac{3[\frac{S^2{}_Q}{Q^2} + \frac{1}{(\sum_{i=1}^{3} n_i^{3/2}P_{wi})^2}\sum n_i{}^2 P_{wi}{}^2 S^2{}_{n_i} + \frac{1}{4S_o{}^2}S^2{}_{S_o}]^{\frac{1}{2}}}{\frac{5}{A}\frac{dA_w}{dy} - \frac{2}{(\sum_{i=1}^{3} n_i^{3/2}P_{wi})}\frac{d}{dy}((\sum_{i=1}^{3} n_i^{3/2}P_{wi})]} \quad (15)$$

Therefore, the constraint of overtopping probability is defined as Eq. (16).

$$\Phi_6 = \left|\frac{f}{Z} - \frac{3[\frac{S^2{}_Q}{Q^2} + \frac{1}{(\sum_{i=1}^{3} n_i^{3/2}P_{wi})^2}\sum n_i P_{wi}{}^2 S^2{}_{n_i} + \frac{1}{4S_o{}^2}S^2{}_{S_o}]^{\frac{1}{2}}}{\frac{5}{A}\frac{dA_w}{dy} - \frac{2}{(\sum_{i=1}^{3} n_i^{3/2}P_{wi})}\frac{d}{dy}((\sum_{i=1}^{3} n_i^{3/2}P_{wi})]}\right| - \varepsilon \leq 0 \quad (16)$$

*2.4. Objective function*

The first step in channel design is determining its optimal dimensions to transfer the flow discharge with the lowest construction cost. In this research, the total cost of constructing one meter of canal includes the costs of excavating (cross-sectional area) and lining the surfaces (perimeter).

$$MinimizeCost = Cost(y, b, z_1, z_2) \quad (17)$$

$$C_1 = C_e + C_L \quad (18)$$

Where $C_e$= Excavation cost is per unit of channel length and $C_L$= cost of channel lining is per unit of channel length.

$$C_e = c_e A_t \quad (19)$$

$c_e$= excavation cost per unit cross-sectional area for a unit length of the channel.

$$C_L = c_l P_t \quad (20)$$

$c_l$= lining costs per unit length of the perimeter.

$$C = c_e A + c_l P \quad (21)$$

C=objective function equal to the total construction cost of the channel consisting of excavation and lining cost.

*2.5. Optimization algorithm*

The SPSA algorithm is a powerful algorithm for the optimization of complex systems that was developed and expanded by SPAL in 1998. Among the features of the SPSA algorithm is that in each optimization iteration, regardless of the number of design variables, it only needs to evaluate the objective function twice. Therefore, the use of this algorithm greatly reduces the volume of calculations and reduces the total optimization time. [36] SPSA algorithm has been effectively used in civil engineering, especially in arch dam optimization [37-43]. The SPSA algorithm is for optimization of unconstrained problems. Therefore, in order to use it in solving bounded problems with unequal constraints, it is necessary to replace the objective function with the pseudo-objective function obtained by using the method of external penalty functions:

$$w(X, r_p) = f(X) + r_p \sum_{j=1}^{m} \max[0, g_j(X)]^2 \Rightarrow f(\bullet) = w(\bullet) + \text{noise} \quad (22)$$

The above steps can be seen in the flowchart of Figure (2).

To use SPSA algorithm for solving constrained problems with inequality constraints $g_j \leq 0$ $(j = 1, \ldots, m)$, it is necessary to replace the quasi-objective function $w$ obtained by the external penalty function method with the objective function $f$ (Eq. 22).

$r_p$ is a penalty multiplier. The flow chart of the SPSA algorithm for the channel optimization problem can be shown in Figure 2.

## 3. Results and discussion

In this article, design steps of the optimal trapezoidal composite channel cross-section have been done by considering the overtopping probability constrained in different values of flow discharge using the SPSA optimization algorithm. The flow discharge values are 10, 50 and 120 $m^3/s$ respectively. These steps are as follows:

- ***Step1:*** Das's method is used to verify the results was used to compare the results of the SPSA algorithm. The values of flow discharge, Manning roughness coefficients, bed slope, freeboard and constant cost values are as follows: $Q = 100\, m^3/s$, $n_1 = 0.015$ ، $n_2 = 0.015$, $c_1 = 0.5$, $c_2 = 0.3$, $c_3 = 0.35$, $c_4 = 0.4$, $f = 0.5m$, $S_0 = 0.0025$

  It should be noted that in the analytical model of Das (2000), only Manning's constraint is specified in the optimization process.

- ***Step2:*** Then, the model was developed with five very important constraints including, uniform flow constraint, Froud number constraint to control the subcritical flow in the channel, maximum velocity constraint to prevent scouring in the channel, minimum velocity constraint to control sedimentation in the channel and limiting the Channel top width were investigated to reduce the cost of the land area in order to provide the optimal shape of the channel section. The aforementioned restrictions are also called definite restrictions.

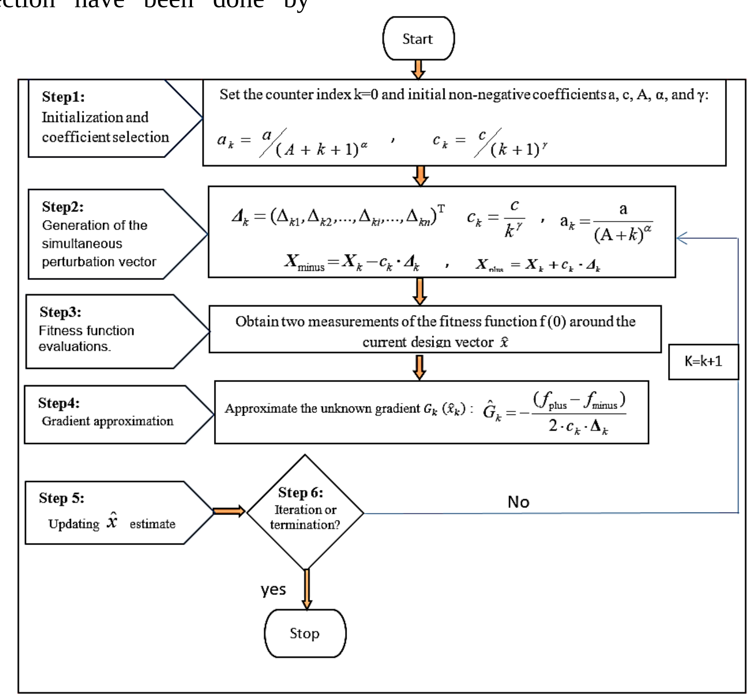


Figure 2. The flow chart of SPSA algorithm [32]

Table 3
Comparison of the optimization results of the trapezoidal composite channel section with the analytical model method of Das (2000)

| Parameters / Model | y (m) | b(m) | Z1 | Z2 | f(m) | COST |
|---|---|---|---|---|---|---|
| Das (2000) | 4.033 | 6.404 | 0.253 | 0.292 | 0.5 | 22.935 |
| SPSA model (uniform flow) | 4.0327 | 6.410 | 0.249 | 0.293 | 0.5 | 22.9354 |
| SPSA model (total constraints) | 4.013 | 6.305 | 0.305 | 0.297 | 0.5 | 22.9444 |

Comparing the results of the current model with the results of the analytical method of Das (2000) shows that the SPSA algorithm has a very good convergence in reaching the desired solution.

- ***Step3:*** After verifying and ensuring the correctness of the model in the first and second steps, the optimal shape of the compound trapezoidal channel section was obtained using the SPSA algorithm with the model of Das (2007), which input data of flow discharge values, roughness coefficients Manning, bed slope, freeboard and cost values in the validation section are as follows:
$Q = 120\, {m^3}/{s}$, $n_1 = 0.033$ ، $n_2 = 0.028$, $n_3 = 0.023$, $c_1 = 0.5$, $c_2 = 0.3$, $c_3 = 0.35$, $c_4 = 0.4$, $f = 0.5m$, $S_0 = 0.0025$
At this step, the values of the standard deviation of flow discharge, roughness coefficients and the bed slope of channel of the model (Das, 2007) are as follows:
$S_Q = 51.89414$, $S_{S_0} = 0.00356$, $S_{n_1} = 0.00572$, $S_{n_2} = 0.00445$, $S_{n_3} = 0.00356$
In Table 4, the comparison of the results obtained from the analytical model of Das (2007) with the results related to the optimization of the cross-section of the composed trapezoidal channel by considering Manning's constraint and the overtopping probability constraint corresponding to the minimum cost shows that the amount of cost using of the SPSA algorithm in similar optimization conditions is less than the analytical model of Das (2007).
- ***Step4:*** the optimization is considered with the overtopping probability by taking into account all the constraints to optimize the cross-section of the composed trapezoidal channel. that the input values are the same as the values of the third step and the optimal design is based on the flow discharge values of 10, 50, and 120 (m^3/s).
In this study, the optimization method was repeated by changing the flow discharge. The obtained results are summarized with tables and graphs.

Table 4
Comparison of the optimization results of the trapezoidal composite channel section with the analytical model method of Das (2007) by considering the possibility of flooding with the SPSA algorithm.

| | COST | P | $S_y$ |
|---|---|---|---|
| Das (2000) | 4.033 | 6.404 | 0.253 |
| SPSA model (uniform flow) | 4.0327 | 6.410 | 0.249 |
| SPSA model (total constraints) | 4.013 | 6.305 | 0.305 |

Tables 5, 6 and 7, show the optimal values of flow depth, channel bed width, left and right-side slope of the channel, flow velocity, Froude number and channel construction cost at different overtopping probability values. The flow discharge values in this paper are 10, 50 and 120 m3/s and the freeboard value are 0.5 meters.

Table 5
Optimal values of design variables, constraints and objective function using SPSA algorithm for flow discharge Q=10 $m^3/s$

| p | y(m) | b(m) | $z_1$ | $z_2$ | f | V(m/s) | Fr | Cost |
|---|---|---|---|---|---|---|---|---|
| 0.01 | 1.756 | 2.167 | 0.613 | 0.597 | 0.5 | 1.763 | 0.490 | 6.565 |
| 0.05 | 1.834 | 2.119 | 0.502 | 0.564 | 0.5 | 1.762 | 0.476 | 6.491 |
| 0.1 | 1.887 | 2.092 | 0.443 | 0.534 | 0.5 | 1.758 | 0.467 | 6.456 |
| 0.15 | 1.928 | 2.069 | 0.438 | 0.484 | 0.5 | 1.754 | 0.460 | 6.436 |
| 0.2 | 1.977 | 2.048 | 0.385 | 0.469 | 0.5 | 1.749 | 0.452 | 6.418 |
| 0.25 | 2.036 | 2.045 | 0.334 | 0.426 | 0.5 | 1.742 | 0.440 | 6.400 |
| 0.3 | 2.084 | 2.033 | 0.305 | 0.397 | 0.5 | 1.736 | 0.432 | 6.395 |
| 0.35 | 2.124 | 2.031 | 0.291 | 0.358 | 0.5 | 1.730 | 0.424 | 6.392 |
| 0.4 | 2.175 | 2.037 | 0.253 | 0.329 | 0.5 | 1.722 | 0.415 | 6.394 |
| 0.45 | 2.199 | 2.044 | 0.261 | 0.288 | 0.5 | 1.718 | 0.410 | 6.395 |
| 0.5 | 2.238 | 2.057 | 0.217 | 0.279 | 0.5 | 1.711 | 0.402 | 6.403 |

Table 6
Optimal values of design variables, constraints and objective function using SPSA algorithm for flow discharge Q=50 $m^3/s$

| p | y(m) | b(m) | $z_1$ | $z_2$ | f | V(m/s) | Fr | Cost |
|---|---|---|---|---|---|---|---|---|
| 0.01 | 3.234 | 4.043 | 0.529 | 0.589 | 0.5 | 2.642 | 0.537 | 15.847 |
| 0.05 | 3.333 | 3.945 | 0.520 | 0.527 | 0.5 | 2.636 | 0.527 | 15.798 |
| 0.1 | 3.427 | 3.838 | 0.494 | 0.504 | 0.5 | 2.629 | 0.519 | 15.774 |
| 0.15 | 3.495 | 3.795 | 0.454 | 0.493 | 0.5 | 2.625 | 0.512 | 15.751 |
| 0.2 | 3.561 | 3.742 | 0.442 | 0.467 | 0.5 | 2.619 | 0.506 | 15.743 |
| 0.25 | 3.614 | 3.733 | 0.412 | 0.450 | 0.5 | 2.615 | 0.500 | 15.731 |
| 0.3 | 3.798 | 3.730 | 0.351 | 0.355 | 0.5 | 2.596 | 0.478 | 15.729 |
| 0.35 | 3.820 | 3.738 | 0.361 | 0.325 | 0.5 | 2.593 | 0.475 | 15.735 |
| 0.4 | 3.893 | 3.748 | 0.321 | 0.306 | 0.5 | 2.585 | 0.467 | 15.750 |
| 0.45 | 3.941 | 3.798 | 0.275 | 0.294 | 0.5 | 2.579 | 0.460 | 15.763 |
| 0.5 | 3.981 | 3.817 | 0.278 | 0.258 | 0.5 | 2.572 | 0.454 | 15.780 |

Table 7
Optimal values of design variables, constraints and objective function using SPSA algorithm for flow discharge Q=120 $m^3/s$

| p | y(m) | b(m) | $z_1$ | $z_2$ | f | V(m/s) | Fr | Cost |
|---|---|---|---|---|---|---|---|---|
| 0.01 | 4.306 | 6.450 | 0.465 | 0.444 | 0.5 | 3.314 | 0.566 | 26.763 |
| 0.05 | 4.348 | 6.439 | 0.443 | 0.428 | 0.5 | 3.313 | 0.562 | 26.733 |
| 0.1 | 4.386 | 6.387 | 0.436 | 0.420 | 0.5 | 3.311 | 0.559 | 26.722 |
| 0.15 | 4.500 | 6.170 | 0.430 | 0.418 | 0.5 | 3.302 | 0.553 | 26.717 |
| 0.2 | 4.561 | 6.084 | 0.426 | 0.405 | 0.5 | 3.297 | 0.548 | 26.714 |
| 0.25 | 4.666 | 5.947 | 0.402 | 0.400 | 0.5 | 3.289 | 0.541 | 26.710 |
| 0.3 | 4.808 | 5.898 | 0.354 | 0.360 | 0.5 | 3.278 | 0.528 | 26.705 |
| 0.35 | 4.861 | 5.924 | 0.328 | 0.338 | 0.5 | 3.273 | 0.522 | 26.707 |
| 0.4 | 4.913 | 5.942 | 0.317 | 0.308 | 0.5 | 3.267 | 0.517 | 26.718 |
| 0.45 | 4.946 | 5.984 | 0.286 | 0.300 | 0.5 | 3.264 | 0.512 | 26.725 |
| 0.5 | 4.956 | 6.021 | 0.267 | 0.298 | 0.5 | 3.262 | 0.510 | 26.728 |

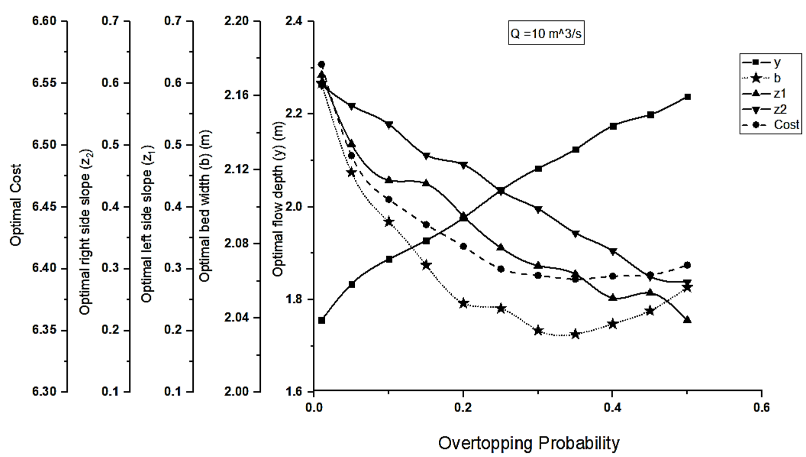


Figure 3. Results of optimization of composed trapezoidal cross-section using SPSA algorithm for flow discharge of 10 $m^3/s$

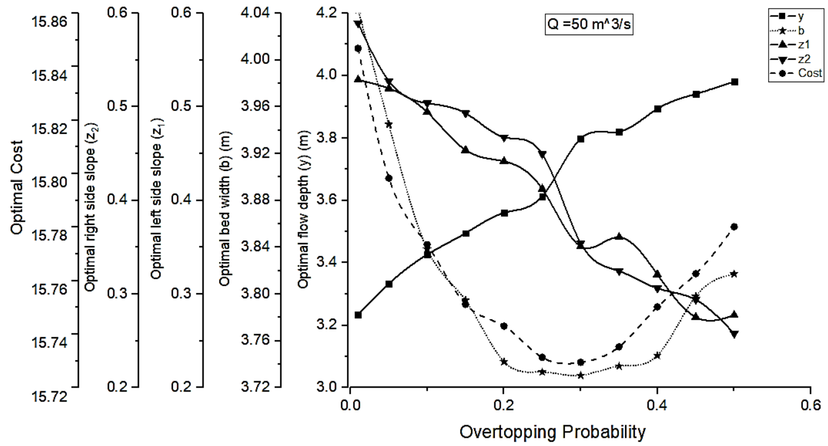


Figure 5: Results of optimization of composed trapezoidal cross-section using SPSA algorithm for flow discharge of 50 $m^3/s$

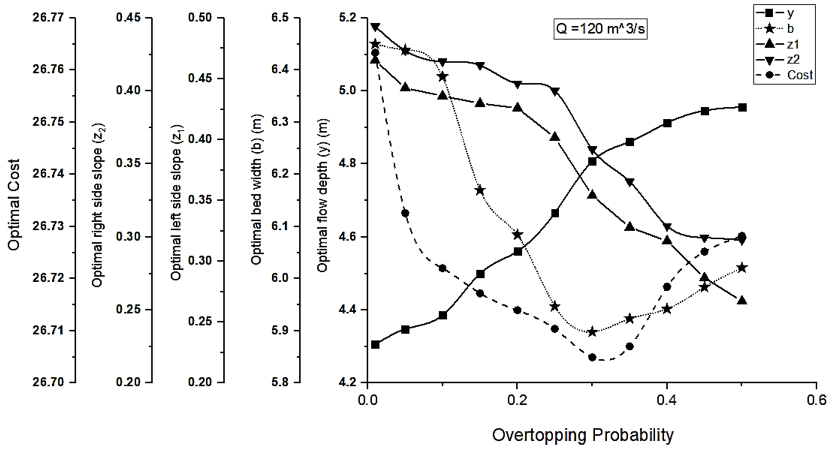


Figure 6. Results of optimization of composed trapezoidal cross-section using SPSA algorithm for flow discharge of 120 $m^3/s$

Figures 3 to 6 show the changes in the optimal values of flow depth, channel bed width, left and right-side slope of the channel, flow velocity, Froud number and channel construction cost with overtopping probability values using the SPSA algorithm. The result show that with increasing probability values, the flow depth increases and, in contrast, with the left and right-side slopes of the channel section, the flow velocity and Froud number decrease. These changes are also valid for all discharge values of 10, 50 and 120 m3/s. The values of the channel bed width and channel construction cost are first decreasing and then increasing. So that initially, with increasing overtopping probability values, the channel bottom width and construction cost decrease, but at a discharge of 10 m3/s from an overtopping probability value of 0.35 and above, and for flow discharge values of 50 and 120 m3/s from an overtopping probability value of 0.3 and above, the channel bottom width and construction cost increase with increasing overtopping probability values.

In Figure 7, the graph shows the changes in flow discharge values with overtopping probability values at flow values of 10, 50, and 120 $m^3/s$.

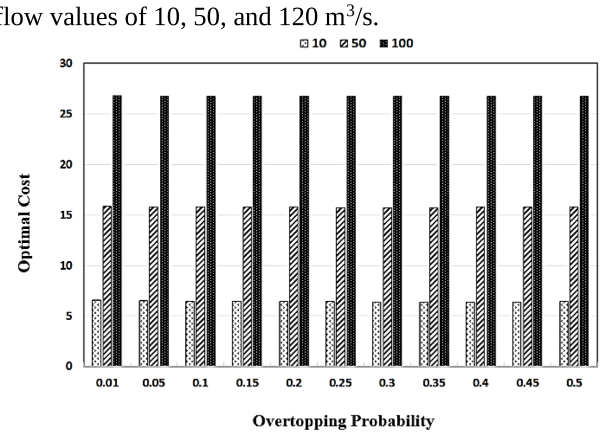


Figure 7: Comparison of the cost of constructing a compound trapezoidal canal for different overtopping probability values and discharge values of 10, 50, and 120 ($m^3/s$).

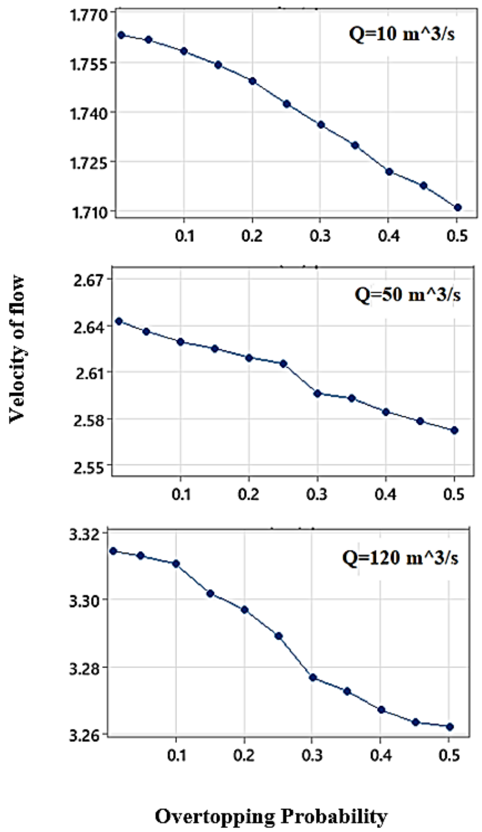


Figure 8: Optimal flow velocity values- overtopping probability relationship for different flow discharge values

Figure 8, shows the optimal values of flow velocity in a composed trapezoidal channel for different overtopping probability values (from 0.01 to 0.5) and flow discharge values of 10, 50, and 120 $m^3$/s. with the flow velocity decreasing as overtopping probability values increase.

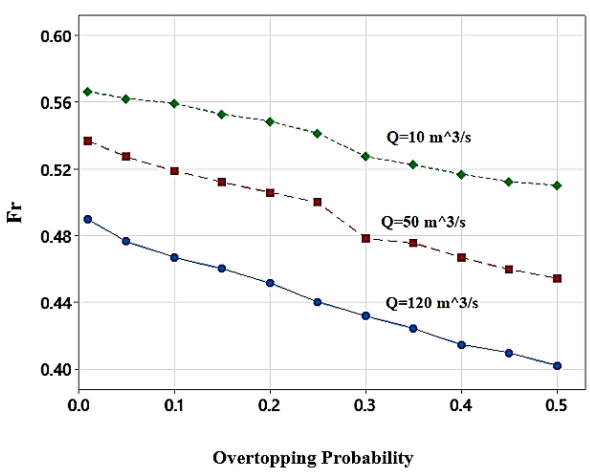


Figure 9: Optimal Froud values - overtopping probability relationship for different flow discharge values

Figure (8) shows the values of the Froude number in a composed trapezoidal channel for different overtopping probability values (from 0.01 to 0.5) and flow discharge values of 10, 50, and 120 $m^3$/s. As the overtopping probability values increase, the Froude number decreases.

## 4. Conclusion

In this study, the effect of uncertainty in the optimal design of a composed trapezoidal channel is presented. The effect of discharge is investigated by considering different values in these models on the design variables, constraints, objective function and geometric parameters and channel design, at different overtopping probability values. The results show that at a certain flow discharge value, with increasing overtopping probability values, the flow depth increases, but the left and right-side slopes, flow velocity and Froude number decrease. The changes in overtopping probability values are somewhat different with the channel bed width and cost function, so that the channel bed width and channel construction cost first decrease and then increase with increasing probability values. Accordingly, for a flow discharge of 10 m3/s and for an overtopping probability value of 0.35 and above, and for flow discharge values of 50 and 120 m3/s from an overtopping probability value of 0.3 and above, the channel bed width and construction cost increase with increasing overtopping probability values.

## References

[1] Te Chow, Ven. Open channel hydraulics. 1959. New York: McGraw-Hill.

[2] Trout, Thomas J. "Channel design to minimize lining material costs." Journal of the Irrigation and Drainage Division 108.4 (1982): 242-249. https://doi.org/10.1061/JRCEA4.0001392

[3] Guo, Chwen-Yuan, and William C. Hughes. "Optimal channel cross section with freeboard." Journal of irrigation and drainage engineering 110.3 (1984): 304-314. https://doi.org/10.1061/(ASCE)0733-9437(1984)110:3(304)

[4] Monadjemi, Parviz. "General formulation of best hydraulic channel section." Journal of Irrigation and Drainage Engineering 120.1 (1994): 27-35. https://doi.org/10.1061/(ASCE)0733-9437(1994)120:1(27)

[5] Swamee, Prabhata K. "Optimal irrigation canal sections." Journal of Irrigation and Drainage Engineering 121.6 (1995): 467-469. https://doi.org/10.1061/(ASCE)0733-9437(1995)121:6(467)

[6] Swamee, Prabhata K., Govinda C. Mishra, and Bhagu R. Chahar. "Comprehensive Design of Minimum Cost IrrigationCanal Sections." Journal of irrigation and drainage engineering 126.5 (2000): 322-327. https://doi.org/10.1061/(ASCE)0733-9437(2000)126:5(322)

[7] Swamee, Prabhata K., Govinda C. Mishra, and Bhagu R. Chahar. "Minimum cost design of lined canal sections." Water Resources Management 14 (2000): 1-12. https://doi.org/10.1023/A:1008198602337

[8] Swamee, Prabhata K., Govinda C. Mishra, and Bhagu R. Chahar. "Design of minimum seepage loss canal sections with drainage layer

at shallow depth." Journal of irrigation and drainage engineering 127.5 (2001): 287-294. https://doi.org/10.1061/(ASCE)0733-9437(2001)127:5(287)

[9] Swamee, Prabhata K., Govinda C. Mishra, and Bhagu R. Chahar. "Design of minimum water-loss canal sections." Journal of Hydraulic Research 40.2 (2002): 215-220. https://doi.org/10.1080/00221680209499864

[10] Swamee, Prabhata K., Govinda C. Mishra, and Bhagu R. Chahar. "Optimal design of transmission canal." Journal of Irrigation and Drainage Engineering 128.4 (2002): 234-243. https://doi.org/10.1061/(ASCE)0733-9437(2002)128:4(234)

[11] Babaeyan-Koopaei, K., E. M. Valentine, and D. C. Swailes. "Optimal design of parabolic-bottomed triangle canals." Journal of Irrigation and Drainage Engineering 126.6 (2000): 408-411. https://doi.org/10.1061/(ASCE)0733-9437(2000)126:6(408)

[12] Das, Amlan. "Optimal channel cross section with composite roughness." Journal of irrigation and drainage engineering 126.1 (2000): 68-72. https://doi.org/10.1061/(ASCE)0733-9437(2000)126:1(68)

[13] Jain, Ashu, Rajib Kumar Bhattacharjya, and Srinivasulu Sanaga. "Optimal design of composite channels using genetic algorithm." Journal of Irrigation and Drainage Engineering 130.4 (2004): 286-295. https://doi.org/10.1061/(ASCE)0733-9437(2004)130:4(286)

[14] Bhattacharjya, Rajib Kumar. "Optimal design of open channel section considering freeboard." ISH Journal of Hydraulic Engineering 11.3 (2005): 141-151. https://doi.org/10.1080/09715010.2005.10514808

[15] Bhattacharjya, Rajib Kumar. "Optimal design of open channel section incorporating critical flow condition." Journal of irrigation and drainage engineering 132.5 (2006): 513-518. https://doi.org/10.1061/(ASCE)0733-9437(2006)132:5(513)

[16] Bhattacharjya, Rajib Kumar, and Mysore G. Satish. "Optimal design of a stable trapezoidal channel section using hybrid optimization techniques." Journal of irrigation and drainage engineering 133.4 (2007): 323-329. https://doi.org/10.1061/(ASCE)0733-9437(2007)133:4(323)

[17] Das, Amlan. "Flooding probability constrained optimal design of trapezoidal channels." Journal of irrigation and drainage engineering 133.1 (2007): 53-60. https://doi.org/10.1061/(ASCE)0733-9437(2007)133:1(53)

[18] Bhattacharjya, Rajib Kumar, and Mysore Satish. "Flooding probability-based optimal design of trapezoidal open channel using freeboard as a design variable." Journal of irrigation and drainage engineering 134.3 (2008): 405-408. https://doi.org/10.1061/(ASCE)0733-9437(2008)134:3(405)

[19] Reddy, M. Janga, and S. Adarsh. "Overtopping probability constrained optimal design of composite channels using swarm intelligence technique." Journal of irrigation and drainage engineering 136.8 (2010): 532-542. https://doi.org/10.1061/(ASCE)IR.1943-4774.0000217

[20] Das, Amlan. "Cost and flooding probability minimization based design of HBPS channel." Water resources management 24 (2010): 193-238. https://doi.org/10.1007/s11269-009-9444-z.

[21] Janga Reddy, M., and Sankaran Adarsh. "Chance constrained optimal design of composite channels using meta-heuristic techniques." Water resources management 24 (2010): 2221-2235. https://doi.org/10.1007/s11269-009-9548-5

[22] Adarsh, S. "Modeling parametric uncertainty in optimal open channel design using FORM-PGSL coupled approach." Stochastic environmental research and risk assessment 26 (2012): 709-720. https://doi.org/10.1007/s00477-011-0539-8

[23] Sankaran, Adarsh, and Janga Reddy Manne. "Probabilistic multi-objective optimal design of composite channels using particle swarm optimization." Journal of hydraulic research 51.4 (2013): 459-464. https://doi.org/10.1080/00221686.2013.777372

[24] Easa, Said M. "New and improved channel cross section with piecewise linear or smooth sides." Canadian Journal of Civil Engineering 38.6 (2011): 690-697. https://doi.org/10.1139/l11-037

[25] Easa, Said M., and Ali R. Vatankhah. "New open channel with elliptic sides and horizontal bottom." KSCE Journal of Civil Engineering 18 (2014): 1197-1204. https://doi.org/10.1007/s12205-014-0559-2

[26] Han, Yan-Cheng. "Horizontal bottomed semi-cubic parabolic channel and best hydraulic section." Flow Measurement and Instrumentation 45 (2015): 56-61. https://doi.org/10.1016/j.flowmeasinst.2015.04.001

[27] Han, Yan-Cheng, and Said M. Easa. "New and improved three and one-third parabolic channel and most efficient hydraulic section." Canadian Journal of Civil Engineering 44.5 (2017): 387-391. https://doi.org/10.1139/cjce-2016-0535

[28] Orouji, Hossein, et al. "Shuffled frog-leaping algorithm for optimal design of open channels." Journal of irrigation and drainage engineering 142.10 (2016): 06016008. https://doi.org/10.1061/(ASCE)IR.1943-4774.0001059

[29] Roushangar, Kiyoumars, et al. "A cost model with several hydraulic constraints for optimizing in practice a trapezoidal cross section." Journal of Hydroinformatics 19.3 (2017): 456-468. https://doi.org/10.2166/hydro.2017.081

[30] Gupta, Sanjay K., et al. "Fish shoal optimization for identification of the most suitable revetment stone for design of minimum cost earthen canals carrying sediment-laden flow." ISH Journal of Hydraulic Engineering 24.2 (2018): 172-189. https://doi.org/10.1080/09715010.2017.1402211

[31] Farzin, Saeed, and Mahdi Valikhan Anaraki. "Optimal construction of an open channel by considering different conditions and uncertainty: application of evolutionary methods." Engineering Optimization 53.7 (2021): 1173-1191. https://doi.org/10.1080/0305215X.2020.1775825

[32] Pourbakhshian, Somayyeh, and Majid Pouraminian. "Analytical models for optimal design of a trapezoidal composite channel cross-section." Civil and Environmental Engineering Reports 31.1 (2021). https://doi.org/10.2478/ceer-2021-0009

[33] Pourbakhshian, S., and P. Fasih. "OPTIMIZATION OF COMPOSI TE PARABOLIC CHANNEL CROSS-SECTION BASED ON CH ANGES IN DISCHARGE AND FREEBOARD." Sharif Journal of Civil Engineering 38.1.2 (2022): 155-167. https://doi.org/10.24200/J30.2022.58880.3011

[34] Farzin, Saeed, et al. "Optimization Modelling of Irrigation Channel Considering Flooding Conditions and Uncertainty." Irrigation and Water Engineering 9.1 (2018): 46-60. https://doi.org/10.1080/0305215X.2020.1775825

[35] Horton, Robert E. "Separate roughness coefficients for channel bottom and sides." Eng. News-Rec 111.22 (1933): 652-653.

[36] Spall, James C. "An overview of the simultaneous perturbation method for efficient optimization." Johns Hopkins apl technical digest 19.4 (1998): 482-492.

[37] Seyedpoor, Seyed Mohammad, et al. "Optimal design of arch dams subjected to earthquake loading by a combination of simultaneous perturbation stochastic approximation and particle swarm algorithms." Applied Soft Computing 11.1 (2011): 39-48. https://doi.org/10.1016/j.asoc.2009.10.014

[38] Pourbakhshian, S., and M. Ghaemain. "Shape optimization of arch dams using sensitivity analysis." KSCE Journal of Civil Engineering 20 (2016): 1966-1976. https://doi.org/10.1007/s12205-015-0135-4

[39] Pourbakhshian, S., M. Ghaemain, and A. Jogatae. "The shape optimization of concrete arch dams considering stage construction."

Scientia Iranica 23.1 (2016): 21-35. https://doi.org/10.24200/SCI.2016.2094

[40] Pouraminian, Majid, and Somayeh Pourbakhshian. "SPSA Algorithm based optimum design of longitudinal section of bridges." Indian Journal of Science and Technology 7.9 (2014): 1327-1332. https://doi.org/10.17485/ijst/2014/v7i9/59472

[41] Pouraminian, Majid, and Somayyeh Pourbakhshian. "Shape optimization of concrete arch dams with SPSA algorithm and post-analysis of pareto front for selection the best alternatives." Journal of Hydraulic Structures 6.3 (2020): 59-82. https://doi.org/10.22055/JHS.2020.33293.1137

[42] Pouraminian, Majid, and Mohsen Ghaemian. "Shape optimisation of concrete open spandrel arch bridges." Građevinar 67.12. (2015): 1177-1185. https://doi.org/10.14256/JCE.1223.2015

[43] Rahgozar, Navid, Majid Pouraminian, and Nima Rahgozar. "Structural optimization of vertical isolated rocking core-moment frames." Journal of Vibration and Control 29.15-16 (2023): 3451-3460. https://doi.org/10.1177/10775463221096882